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Automatic modulation classification for multi-criteria generic channel equalization

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Abstract—In this paper, we study blind equalization techniques to mitigate inter-symbol interference (ISI), and mainly, we are focused on generic blind equalizer (GBE). A GBE has no prior information about the transmission channel or the used constellation. To solve this challenge, a joint generic blind equalizer, based on a new multi-criteria cost function and automatic modulation classification (AMC) is proposed. The new multi-criteria cost function is based on the probability density fitting (PDF) and the k-nearest neighbor (KNN) algorithm is used for the AMC stage. Thus, using a neural architecture, the new criterion is demonstrated in its linear and nonlinear context. Simulation results support our claims with Quadrature Amplitude Modulation (QAM) transmitted signals in single input single output (SISO) communication system and they show a better performance in terms of mean square error (MSE) and symbol error rate (SER) compared to other GBE from the literature.

Index Terms—Blind equalization, AMC, pdf, neural network.

I. Introduction

Blind equalization is a very interesting research topic aiming to develop effective and low-complexity algorithms that reduce inter-symbol interference (ISI) and avoid bandwidth waste resulting from training data. Generic blind equalization is needed when constructing intelligent receivers that do not know the channel and the transmitted modulation type. To estimate the transmitted signal, the receiver has to blindly equalize the channel, then, it has to reliably classify the transmitted modulation by considering an adequate AMC technique.

Although massive access has a great interest in the Internet of Things (IoT) communication system, the SISO model is well used and we have shown our contribution with a SISO model first.

In the literature, among the works addressing joint blind equalization and AMC, we can refer to [1]. In this paper, the authors suggested using different equalization branches, each adapted to a specific constellation. This yields a complex architecture system where the filter providing the smallest cost function indicates the right constellation. In [2], the constant modulus algorithm (CMA) was used to produce a generic equalizer with a normalized radius. They used the CMA cost function relative to the quadrature phase-shift keying (QPSK) constellation to equalize any transmitted constellation belonging to the phase-shift keying

modulation (PSK) or the QAM modulation. As a result, the equalizer output is symbols belonging to the transmitted constellation but compressed in the unit radius. After the classification by the amplitude of the equalized signal characteristic function (CF), the equalized signal is multiplied by a scaling factor to restore the emitted constellation amplitudes before deciding and exploiting the transmitted data.

Moreover in [3], authors follow the same idea as [2] but instead of using the CMA criterion, they suggested considering the PDF with the multi-modulus stochastic quadratic distance (MSQD- ℓ_p) criterion [4]. In this case, during the convergence phase, the equalizer fits the PDF of the received symbols to match with the QPSK constellation PDF regardless of the emitted modulation. Concerning the AMC, three methods were proposed that detect the modulation which has the PDF that best fits, in some sense, that of the equalizer output.

In all these mentioned works, AMC is applied after equalization of the received signal. However, this paper proposes a new generic equalizer architecture that combines jointly blind equalization and AMC.

In [3], generic equalizer MSQD- ℓ_2^{gen} based on multi-modulus stochastic quadratic distance criterion outperforms generic equalizer CMA $_{gen}$ based on constant modulus algorithm criterion in [2]. For this reason, we have considered MSQD- ℓ_2^{gen} as a benchmark in its linear and neural version. For the equalization step, we propose a new multi-criteria generic cost function based on the PDF with the MSQD- ℓ_p criterion since it outperforms those based on high order statistical properties criterion [5]. In addition, we implemented the proposed criterion in linear and nonlinear case banks complex-valued neural networks (CVNN). For the AMC step, in the literature, various techniques for modulation classification are considered such us Maximum-likelihood [6] [7] and order cyclic cumulants [8] [9]. In this work, we apply the KNN algorithm with the fourth-order cumulants [10] [11].

The main contributions of this paper are :

- Proceeding blind equalization and AMC simultaneously.
- Derivation of a new multi-criteria cost function for a generic equalization.

The rest of the paper is organized as follows. In Section II, we introduce the system model. In Section III, we detail our multi-criteria generic cost function based on the MSQD- ℓ_p algorithm in its linear and neural versions. Simulation results are expounded in Section IV. Finally, Section V concludes our work.

II. System model

In this section, we introduce the baseband model of a transmission system with a generic equalizer in its linear and nonlinear forms and AMC as it is Fig.1, where $s(n)_{n \in \mathbb{Z}}$ is an M-QAM transmitted symbol, at time n , through a multi-path channel with a finite impulse response (FIR) with length L_h , denoted $h = [h_0, h_1, \dots, h_{L_h-1}]$.

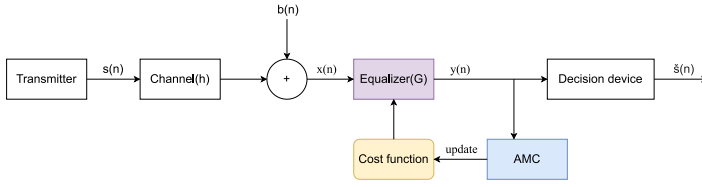


Fig. 1: Transmission system baseband model with a generic equalizer and AMC.

$b(n)$ is an independent and identically distributed (i.i.d) additive white Gaussian noise, $x(n)$ is the input of the equalizer, G is the equalizer function such as :

- Linear context : $G = [w_0, w_1, \dots, w_{L_w-1}]^T$ is the equalizer impulse response, with length L_w and $y(n)$ is the equalized signal at time n , $y(n) = \sum_{i=0}^{L_w-1} w_i x(n-i) = w^T x(n)$ and $x(n) = [x(n), x(n-1), \dots, x(n-L_{w+1})]^T$ where $x(n) = \sum_{i=0}^{L_h-1} h_i s(n-i) + b(n)$.
- Nonlinear context: G is a neural network that will be detailed in section III.B.

III. Equalizer model

In this section, two contributions are proposed. The first one is a generic linear multi-criteria equalizer, based on a simultaneous mixture of the AMC and blind equalization. The second contribution is a generic nonlinear multi-criteria equalizer based also on a simultaneous mixture of blind equalization and AMC and using a neural network. The new multi-criteria cost function for the equalization part and the algorithm used for the AMC for each contribution will be detailed.

A. Multi-criteria MSQD- $\ell_{p_{gen}}$ (MC-MSQD- $\ell_{p_{gen}}$) in linear case

The equalizers whose criterion exploits the PDF outperform the equalizers whose criterion exploits the high-order statistical properties [12]. Moreover, decomposing the equalization criterion into an in-phase component and a quadrature component is more efficient than processing two components together, with respect

to a phase shift introduced by the transmission channel [4]. These two reasons lead us to propose a new multi-criteria generic equalization based on the MSQD- ℓ_p algorithm [4]. Specially for $p = 2$ the MSQD- ℓ_2 has the following cost function :

$$J_{MSQD-\ell_2}(w) = -\frac{1}{N_s} \sum_{k=1}^{N_s} K_\sigma(|y_r|^2 - |s_r(k)|^2) - \frac{1}{N_s} \sum_{k=1}^{N_s} K_\sigma(|y_i|^2 - |s_i(k)|^2), \quad (1)$$

where N_s is the number of complex symbols in the considered constellation and K_{σ_0} is a Gaussian Kernel with zero mean and variance σ_0 which is referred to Kernel width. y_r and y_i and s_r and s_i are the real and imaginary parts of the equalized output and the transmitted constellation, respectively.

The equalizers based on PDF fitting aim to minimize a specific distance between the data distribution at the output of the equalizer and a target distribution that corresponds to the transmitted constellation one. Since, in our case, we assume that we do not know the transmitted constellation, we defined a new multi-criteria generic cost function as a sum of several variations of equation (1), one for each constellation order multiplied by an updated penalty factor. So, the criterion that we propose is :

$$J_{MC-MSQD-\ell_{2_{gen}}}(w) = \sum_{m=1}^C \alpha_m J_{MSQD-\ell_{2_m}}(w), \quad (2)$$

where C is the number of the considered constellations orders. The penalty factor α_m in (2) is updated in each iteration according to the following expression such that we reach, over the iterations, the transmitted constellation cost function :

$$\alpha_m = \text{sigmoid}(\text{class}_m) \quad (3)$$

where class_m is the number of times where the m^{th} constellation is successful in the classification result after the classification step.

The equalizer coefficients are updated using the stochastic gradient descent (SGD) algorithm as:

$$w(n+1) = w(n) - \mu \nabla_w J_{MSQD-\ell_{2_{gen}}}(w) = w(n) - \mu \sum_{m=1}^C \alpha_m \nabla_w J_{MSQD-\ell_{2_m}}(w), \quad (4)$$

where μ is the step size.

In the sequel, we will focus on the explicit expression of (2). For this purpose, we begin by calculating the gradient of (1) which leads to that of (2).

On the other hand, $y(n) = w(n)^T x(n)$ can be expressed as $y(n) = [w_r^T x_r(n) - w_i^T x_i(n)] + j[w_r^T x_i(n) + w_i^T x_r(n)]$. It results that : $\partial y(n)/\partial w_r = x(n)$ and $\partial y(n)/\partial w_i = jx(n)$. Consequently, the derivative of $J_{MSQD-\ell_2}(w)$ with respect to

the equalizer weights is :

$$\begin{aligned}
\nabla_w J_{MSQD-\ell_2}(w) &= \frac{\partial J_{MSQD-\ell_2}(w)}{\partial w_r} + j \frac{\partial J_{MSQD-\ell_2}(w)}{\partial w_i} \\
&= \frac{1}{\sqrt{2\pi} N_s \sigma^3} \sum_{k=1}^{N_s} (\text{sign}(y_r(n)) |y_r(n)| \\
&\quad (|y_r(n)|^2 - |s_r(k)|^2) \\
&\quad e^{-(|y_r(n)|^2 - |s_r(k)|^2)^2 / 2\sigma^2} \\
&\quad + j \text{sign}(y_i(n)) |y_i(n)| (|y_i(n)|^2 - |s_i(k)|^2) \\
&\quad e^{-(|y_i(n)|^2 - |s_i(k)|^2)^2 / 2\sigma^2}) x^*(n).
\end{aligned} \tag{5}$$

In order to update the cost function, each equalized symbol is classified using the KNN algorithm with fourth-order cumulant [8] [9] as features. So that we do jointly the equalization and the AMC. The expressions of the p^{th} order cumulant and moment are respectively as follows :

$$\begin{aligned}
C_{pq} &= \text{cum}(x^{p-q}(x^*)^q) \\
\text{and} \\
M_{pq} &= E[x^{p-q}(x^*)^q],
\end{aligned}$$

where $E[\cdot]$ is the expectation operator.

In particular, consider :

$$\begin{aligned}
C_{40} &= M_{40} - 3M_{20}^2 \\
C_{42} &= M_{42} - |M_{20}|^2 - 2M_{21}^2
\end{aligned}$$

as they are more suitable for M-QAM modulations [9]. A baseline containing C_{40} and C_{42} values for noisy symbols according to various values of signal-to-noise ratio (SNR) and belonging to $\{16, 32, 64, 128, 256\}$ -QAM modulations is prepared.

the classifier follows the following steps :

- Calculate $S_1 = C_{40} + C_{42}$ the sum of the last 1000 equalized symbols.
- Calculate the Euclidean distances between S_1 and each sum in the reference base.
- A set of the k nearest neighbors from the baseline is created.
- Since each neighbor matches a specific constellation order, then we select the order is the most repeated order in the previous set.

Finally, according to the AMC, we update the penalty factors following (3). Then (2) will be updated.

B. Multi-criteria neural network $NNMSQD-\ell_{p_{gen}}$ (MC- $NNMSQD-\ell_{p_{gen}}$)

The neural network used for our equalizer is the complex-valued feed-forward which is trained using the complex back-propagation (CBP) algorithm [13] [14] [15]. This neural network is combined with a KNN classifier as it is illustrated in Fig.2

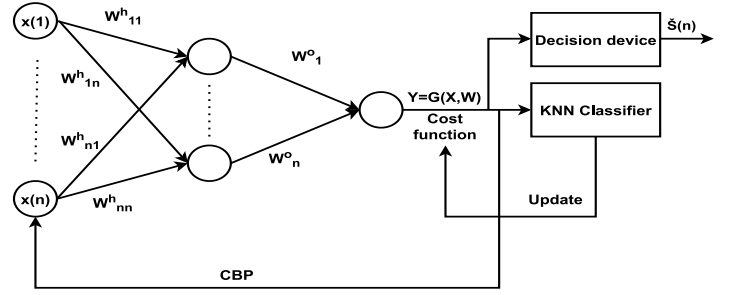


Fig. 2: A complex-valued feed-forward multi-criteria neural network equalizer combined with a KNN classifier.

We assume N_k neurons in the k^{th} layer and we denote by ϕ_j^k and x_j^{k+1} the input and the output of the k^{th} neuron, such that :

$$\phi_j^k = \sum_{i=1}^{N_{k-1}} w_{ij}^k x_i^k + \theta_j^k \quad \text{and} \tag{6}$$

$$x_j^{k+1} = f^k(\text{Re}(\phi_j^k)) + j f^k(\text{Im}(\phi_j^k)), \tag{7}$$

where x_i^k is the k^{th} layer output, w_{ij}^k is the weight between the i^{th} neuron in the k^{th} layer and the j^{th} neuron in the $(k+1)^{th}$ layer, θ_j^k and $f^k(\cdot)$ are the k^{th} layer bias and activation function. In this paper, we consider a complex-valued feed-forward equalizer including two main steps for each iteration. The first step is the forward pass from the input layer to the output layer. This processing consists in computing the network output by applying an activation function on the weighted sum in each layer. The second step is the CBP to update the synaptic weights according to our equalizer criterion. The network includes three layers: input, hidden, and output layers. We assume in this paper that the activation function is as in [16] :

$$f(x) = x + \alpha * \sin(\pi * x),$$

where π is a mathematical constant approximately equal to 3.14159, α a positive number between 0 and 1 and \sin is the trigonometric function. The cost function for the generic multi-criteria neural network equalizer is the same as that of (2) :

$$J_{MC-NNMSQD-\ell_{2_{gen}}}(w) = \sum_{m=1}^C \alpha_m J_{MSQD-\ell_{2_m}}. \tag{8}$$

The penalty factor α_m is calculated and updated as in the MC- $MSQD-\ell_{2_{gen}}$ algorithm and for each iteration, we follow the same steps. First, we equalize the received symbol, then we classify it, afterward, we update the neural network weights and finally, we update the penalty factors respectively.

The weights of the neural network are updated as:

$$w_{ij}^k(n+1) = w_{ij}^k(n) - \mu \sum_{m=1}^C \frac{\alpha_m \partial J_{MSQD-\ell_{2_m}}}{\partial w_{ij}^k(n)}. \tag{9}$$

IV. Simulation results

In the sequel, we focus on the derivation of (8). We begin by calculating the gradient of (1) which leads to that of (8). To update the output layer weights, we use the same procedure as in [16] for simplification purposes, we introduce Q_r , Q_i and δ_p^o which are expressed as the following :

$$Q_r = \frac{1}{N_s \sqrt{2\Pi\sigma}} \sum_{k=1}^{N_s} e^{-\frac{(|y_r(n)|^2 - |s_r(k)|^2)^2}{2\sigma^2}} \frac{(|y_r(n)|^2 - |s_r(k)|^2)}{\sigma^2},$$

$$Q_i = \frac{1}{N_s \sqrt{2\Pi\sigma}} \sum_{k=1}^{N_s} e^{-\frac{(|y_i(n)|^2 - |s_i(k)|^2)^2}{2\sigma^2}} \frac{(|y_i(n)|^2 - |s_i(k)|^2)}{\sigma^2},$$

$$\delta_p^o = Q_r y_r(n) f'^o(\varphi_r^o) + j Q_i y_i(n) f'^o(\varphi_i^o). \quad (10)$$

The weights of the output layer are updated as :

$$w_j^o(n+1) = w_j^o(n) - \mu \sum_{m=1}^C \alpha_m \delta_{p_m}^o I_{pj}^*, \quad (11)$$

where φ_r^o , φ_i^o , are the real and imaginary inputs of the output layer, and I_{pj} is the output of the hidden layer. $f'^o(\varphi_r^o)$ and $f'^o(\varphi_i^o)$ are the derivatives of the activation function in the output layer applied in the real and imaginary parts of the output layer input.

To update the hidden layer weights, we consider the same reasoning as in [16]. We simplify the final expression by introducing δ_{pj}^h ,

$$\delta_{pj}^h = f_j^{h'}(\varphi_r^h) \Re(\delta_p^o w_j^{o*}) + j f_j^{h'}(\varphi_i^h) \Im(\delta_p^o w_j^{o*}). \quad (12)$$

Similarly, the weights of the hidden layer are updated as :

$$w_{ij}^h(n+1) = w_{ij}^h(n) - \mu \sum_{m=1}^C \alpha_m \delta_{pj_m}^h x_{pi}^*, \quad (13)$$

where φ_r^h , φ_i^h , are the real and imaginary inputs of the hidden layer, and x_{pi} is the input vector of the equalizer. $f_j^{h'}(\varphi_r^h)$ and $f_j^{h'}(\varphi_i^h)$ are the derivatives of the activation function of the hidden layer applied in the real and imaginary parts of the hidden layer input at the j^{th} neuron. AMC process is applied using the same principle as for MC-MSQD- $\ell_{2_{gen}}$.

We have simulated our approach using Matlab 2018. We have considered M=16, 32, 64, 128, and 256 M-QAM constellations. So, the penalty factors are equally initialized to 0.2. An important factor for the convergence of the equalizers is the learning rate μ . It is fixed to $10^{-3}/P_x$, where P_x is the received signal power. The total number of symbols used during the training step is 70000 and the equalizers were trained for epoch numbers equal to 1. We evaluate the performance of the proposed equalizers in terms of mean square error (MSE), symbol error rate (SER), and accuracy rate when using a one typical digital radio channel h_{comp} that was used in [4].

$$h_{comp}(z) = (0.0410 + j0.0109) + (0.0495 + j0.0123)z^{-1} + (0.0672 + j0.0170)z^{-2} + (0.0919 + j0.0235)z^{-3} + (0.7920 + j0.1281)z^{-4} + (0.3960 + j0.0871)z^{-5} + (0.2715 + j0.0498)z^{-6} + (0.2291 + j0.0414)z^{-7} + (0.1287 + j0.0154)z^{-8} + (0.1032 + j0.0119)z^{-9}.$$

MC-MSQD- $\ell_{2_{gen}}$ and MSQD- $\ell_{2_{gen}}$ equalizers size is $L_w = 31$ and their weights are initialized with null values, except the middle element which is set to 1.

Then, we have simulated MC-NNMSQD- $\ell_{2_{gen}}$ and NNMSQD- $\ell_{2_{gen}}$ equalizers with a complex-valued feed-forward neural network using three layers. The number of neurons in the input, the hidden, and the output layers are [$L_i = 15$ (input), $L_h = 9$ (hidden), $L_o = 1$ (output)]. The activation function is $f(x) = x + \alpha \sin(\Pi x)$ where α is fixed to 0.1 and is independent of the modulation size, signal-to-noise ratio and layers order. We have initialized the two matrices of synaptic weights $w^o[L_o, L_h]$ and $w^h[L_h, L_i]$ with small values. Except for $w^o[(L_o + 1)/2, (L_h + 1)/2]$ and $w^h[(L_h + 1)/2, (L_i + 1)/2]$ that were set to 1.5.

Equalizers have the same convergence speed and we compared performances in terms of MSE, SER, and accuracy rate.

Computation burdens for MC-MSQD- $\ell_{2_{gen}}$ and MC-NNMSQD- $\ell_{2_{gen}}$ are defined in Tab I.

Equalizer	X	Exponents
MC-MSQD- $\ell_{2_{gen}}$	L_w	$2N_s$
MC-NNMSQD- $\ell_{2_{gen}}$	$3L_h(L_i + L_o) + 2L_h$	$2N_s$

TABLE I: Computational complexity

Figures (3.a) and (3.b) show the performance of the different equalizers in terms of MSE according to the SNR, when we consider the value of MSE after the convergence of the algorithm. It can be seen that for the considered transmitted constellation, our linear approach is more performant than the linear approach in [4] and its developed neural version. Also, the proposed neural approach MC-NNMSQD- $\ell_{2_{gen}}$ outperforms the MC-MSQD- $\ell_{2_{gen}}$. Figures (4.a) and (4.b) prove that our generic equalizers guarantee interesting performance in terms of SER and it has the best performance for the different constellations.

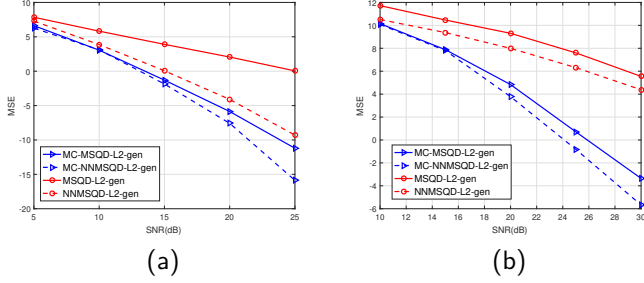


Fig. 3: MSE versus SNR:(a) 16-QAM, (b) 64-QAM.

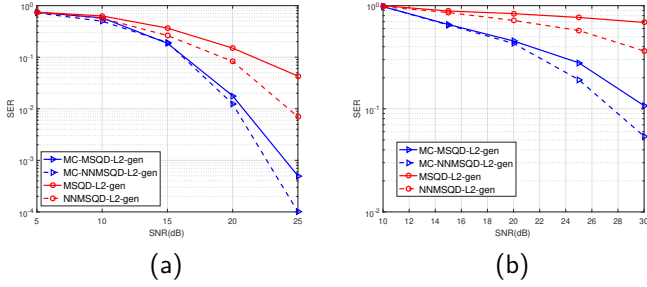


Fig. 4: SER versus SNR:(a)16-QAM, (b)64-QAM.

Figure 5 shows that the classification rate reaches an acceptable

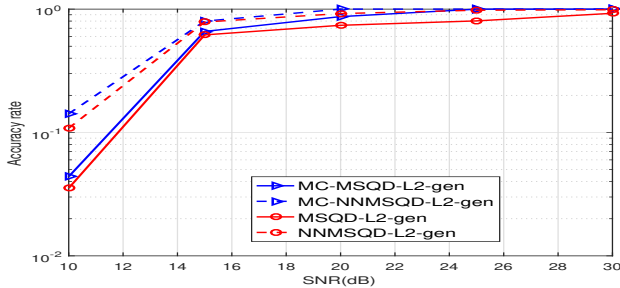


Fig. 5: Accuracy rate for 64-QAM and h_{comp} .

value when the SNR exceeds 15dB. The classification rate is better with our new multi-criteria in its neural and linear versions.

V. Conclusion

In this paper, we have proposed a new blind generic equalization approach. This technique is based on simultaneous joint blind equalization and AMC. For the equalization stage, a new generic multi-criteria cost function is derived. Differently from the work in the literature, where the equalization criterion is static, this new criterion is dynamic and updated during the process. For the AMC stage, the KNN algorithm is used with the fourth-order

cumulant as a feature. The obtained simulation results prove that our proposed techniques lead to outperforming the MSQD- l_{2-gen} in terms of MSE, SER, and accuracy rate in linear and nonlinear form. As a perspective, we aim at generalizing our approach to other constellations and extending this approach for multiple inputs multiple outputs (MIMO) architecture.

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