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# Alternating Hybrid Beamforming Design using GMD decomposition for mmWave MIMO-OFDM Systems

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**Abstract**—Since millimeter wave (mmWave) communications have wideband channels, mmWave signal corruptions increase due to radio-channel frequency selectivity. In this case, the combination of the orthogonal frequency division multiplexing (OFDM) with the mmWave MIMO system is envisioned as a candidate technique to address the degradation of communication. As hybrid analog/digital architecture offers potential energy and spectral efficiency for the mmWave MIMO device. Matrix factorization formulation with singular value decomposition (SVD) is the most used method for designing the hybrid precoder/combiner. However, using SVD decomposition in precoding/combining designing requires power allocation schemes due to the different signal-to-noise ratios (SNRs) of different sub-channels. To achieve a high wireless communications capacity, we propose in this work an alternating minimization algorithm based on a manifold optimization technique using the geometric mean decomposition (GMD) (called MO-AltMin-GMD) to derive unconstrained optimal precoders and combiners from the channel state information (CSI). The main advantage is that the proposed hybrid design avoids any allocation schemes in order to reduce the hybrid architecture complexity. Numerical simulations show that the proposed hybrid design provides high results compared to the existing methods in terms of spectral efficiency.

**Index Terms**—MmWave MIMO OFDM system, hybrid precoder/combiner design, MO-AltMin algorithm, geometric mean decomposition, wideband channels.

## I. INTRODUCTION

With the exponential increase of wireless data traffic, millimeter wave (mmWave) communications continue to attract the extensive interest of researchers to meet the potential capacity requirements of the future wireless communication [1][2]. By using the largely underutilized bandwidths between 30 GHz to 300 GHz, mmWave frequencies can enable the support of ultra-high-speed connections with multiGigabit-per-second (Gbps) data rates in indoor and fixed outdoor scenarios [3]. However, signals in mmWave frequencies are known to suffer from severe propagation loss and hardness to bypass obstacles due to their weak diffractive ability [4][5]. On the other hand, the disadvantage of the small mmWave wavelength enables communication devices to have a compact size with a lot of antennas [6][7]. For this, large-scale antenna arrays at

mmWave transceivers can perform directional beamforming to achieve high beamforming gain for overcoming the severe attenuation of mmWave signals by guaranteeing a sufficient received signal-to-noise ratio (SNR) [8]. Due to the huge computational load and cost, a dedicated radio frequency (RF) chain for each antenna poses major engineering hurdles for realizing the fully digital signal processing in order to obtain high performance in mmWave communication systems. To address the trade-off between the performance system and hardware complexity, the hybrid analog/digital architecture is a more attractive alternative because of its potential energy and spectral efficiency for mmWave MIMO devices by achieving a significant beamforming gain and high diversity order [9][10][11]. As mmWave carrier frequencies have wideband channels, mmWave signal corruptions increase due to radio-channel frequency selectivity [12]. Thus, the using of orthogonal frequency division multiplexing (OFDM) is necessary to convert a frequency-selective mmWave MIMO channel into a parallel collection of frequency-flat sub-channels [13]. Therefore, the combination of OFDM and mmWave MIMO is a promising technology for future wireless communications due to their potential to overcome inter-symbol interference (ISI) effect and increasing the spectral efficiency of multi-user/multi-stream communication [13][14]. More specially, the design of optimal precoders and combiners play the most important role to achieve highly directional beamforming in mmWave communication. When the channel state information (CSI) is known at the transceiver, how to derive unconstrained optimal precoders and combiners from CSI is a critical challenge. In [15], spatially sparse hybrid precoding method is proposed using the truncated singular value decomposition (SVD) of the channel matrix, where the right singular matrix is considered as the unconstrained optimal precoding matrix. Furthermore, the optimal hybrid precoders are found by minimizing the Euclidean norm of the unconstrained optimal precoder and the product of digital baseband precoders with analog precoders that lead to maximizing spectral efficiency. The authors in [16] propose an algorithm to design the optimal

hybrid precoder for the wideband mmWave MIMO-OFDM system by using manifold optimization (MO). In [17], the hybrid precoder design is formulated as a matrix factorization problem, and an alternating minimization-based manifold optimization algorithm (MO-AltMin) is proposed to solve this problem. In the works discussed above, unconstrained optimal precoders and combiners are obtained from the channel's singular value decomposition. However, SVD-based hybrid precoding decomposes the MIMO channel into single input single output (SISO) sub-channels with different gains. As known, using SVD decomposition in precoding design can not achieve the high wireless communications capacity without any power allocation scheme. Nevertheless, a power allocation scheme requires an increased dynamic range for power amplifiers (PAs) which augments the architecture complexity. Since different sub-channels in SVD based hybrid precoding have different SNRs, allocating techniques and coding schemes are certainly required which increases also the transceiver complexity [18]. As a multiple frequency-flat sub-channels generated from OFDM, we adopt in this paper the MO-AltMin algorithm with using the geometric mean decomposition (GMD) (called MO-AltMin-GMD) to derive the unconstrained optimal precoder and combiner of each subcarrier. The advantage of using the GMD decomposition is to convert the mmWave MIMO OFDM channel into sub-channels with the same gain for avoiding allocation schemes and maintain the low hybrid architecture complexity. In addition, using the GMD decomposition can achieve asymptotically optimal capacity and excellent bit error rate performance [19].

The rest of the paper is organized as follows. In Section II, the system and wide mmWave channel models are presented. The formulation of hybrid precoder/combiners problem is described in Section III. Simulation results of the spectral efficiency performance compared with other methods are conducted in Section IV. Finally, concluding remarks are drawn in Section V.

## II. SYSTEM MODEL

We consider a MIMO-OFDM system with hybrid architecture as shown in Fig.1. The transmitter (Tx) is equipped with  $N_{tx}$  transmit antennas and  $N_{RF}^{tx}$  RF chains to simultaneously send  $N_s$  data streams for  $K$  subcarriers in the communication link whereas, the receiver (Rx) has  $N_{rx}$  antennas and  $N_{RF}^{rx}$  RF chains. In the hybrid transceiver architecture, the number of data streams at both Tx and Rx is subject to the constraints  $N_s \leq N_{RF}^{tx} \leq N_{tx}$  and  $N_s \leq N_{RF}^{rx} \leq N_{rx}$ . At the  $k^{th}$  subcarrier, the Tx applies the frequency-selective baseband precoder  $\mathbf{F}_{BB}[k] \in \mathbb{C}^{N_{RF}^{tx} \times N_s}$  followed by the frequency-flat analog domain RF precoder  $\mathbf{F}_{RF} \in \mathbb{C}^{N_{tx} \times N_{RF}^{tx}}$  for enabling the transmission. For fully-connected structure, the  $\mathbf{F}_{RF}$  is implemented using analog phase shifters and its entries are constrained to satisfy  $([\mathbf{F}_{RF}]_{:,i}[\mathbf{F}_{RF}]_{:,i}^H)_{\ell,\ell} = N_{tx}^{-1}$ , where  $(\cdot)_{\ell,\ell}$  denotes the  $\ell^{th}$  diagonal element, i.e., all entries are of equal norm. Therefore, the discrete-time transmitted signal at each  $k$  subcarrier can be expressed as

$$\mathbf{x}[k] = \mathbf{F}_{RF}\mathbf{F}_{BB}[k]\mathbf{s}[k] \quad (1)$$

where  $\mathbf{s}[k] \in \mathbb{C}^{N_s \times 1}$  is vector symbol with  $\mathbb{E}[\mathbf{s}[k]\mathbf{s}^H[k]] = \frac{1}{N_s}\mathbf{I}_{N_s}$ . Due to the hybrid architecture, the  $\mathbf{F}_{RF}$  is commonly applied to all the subcarriers, and the total transmit power constraint is enforced by normalizing  $\mathbf{F}_{BB}[k]$  to satisfy  $\|\mathbf{F}_{RF}\mathbf{F}_{BB}[k]\|_F^2 = N_s$ .

For the sparse nature of the mmWave MIMO channel, we adopt the wideband clustered channel representation that is based on the Saleh-Valenzuela model as [16] to represent the limited paths in the angular domain. Then, the downlink frequency domain channel matrix  $\mathbf{H}[k]$  at  $k^{th}$  subcarrier is expressed as

$$\mathbf{H}[k] = \gamma \sum_{i=0}^{N_{cl}-1} \sum_{l=1}^{N_{ray}} \alpha_{il} \mathbf{a}_{rx}(\phi_{il}^{rx}, \theta_{il}^{rx}) \mathbf{a}_{tx}(\phi_{il}^{tx}, \theta_{il}^{tx})^H e^{-j2\pi ik/K} \quad (2)$$

where  $N_{cl}$  and  $N_{ray}$  indicate the number of clusters and the number of rays in each cluster.  $k \in \{0, 1, \dots, K-1\}$  is the subcarrier index, whereas  $K$  is the total number of subcarriers.  $\alpha_{il}$  is the complex channel gain of  $l^{th}$  ray in the  $i^{th}$  clusters. In this paper, we assume that  $\alpha_{il}$  is independent and identically distributed (i.i.d.) random variables following the complex Gaussian distribution  $\mathcal{CN}(0, \sigma_{\alpha,i}^2)$ , such that  $\sum_{i=1}^{N_{cl}} \sigma_{\alpha,i}^2 = \gamma$ , and  $\gamma = \sqrt{N_{tx}N_{rx}/N_{cl}N_{ray}}$  is the normalization factor to satisfy  $\mathbb{E}[\|\mathbf{H}[k]\|_F^2] = N_{tx}N_{rx}$ . The function  $\mathbf{a}_{tx}(\phi_{il}^{tx}, \theta_{il}^{tx})$  and  $\mathbf{a}_{rx}(\phi_{il}^{rx}, \theta_{il}^{rx})$  are the transmit and receive array response vectors at an azimuth (elevation) angle of  $\phi_{il}^{tx}$  ( $\theta_{il}^{tx}$ ) and  $\phi_{il}^{rx}$  ( $\theta_{il}^{rx}$ ) respectively. Assuming the uniform planar array (UPA) configuration,  $\mathbf{a}_{tx}(\phi_{il}^{tx}, \theta_{il}^{tx})$  and  $\mathbf{a}_{rx}(\phi_{il}^{rx}, \theta_{il}^{rx})$  can be expressed as

$$\begin{aligned} \mathbf{a}_{tx}(\phi_{il}^{tx}, \theta_{il}^{tx}) &= \frac{1}{\sqrt{N_{tx}}} [1, \dots, e^{j\frac{2\pi}{\lambda}d(p \sin(\phi_{il}) \sin(\theta_{il}) + q \cos(\theta_{il}))}, \\ &\quad \dots, e^{j\frac{2\pi}{\lambda}d((\sqrt{N_{tx}}-1) \sin(\phi_{il}) \sin(\theta_{il}) + (\sqrt{N_{tx}}-1) \cos(\theta_{il}))}] \\ \mathbf{a}_{rx}(\phi_{il}^{rx}, \theta_{il}^{rx}) &= \frac{1}{\sqrt{N_{rx}}} [1, \dots, e^{j\frac{2\pi}{\lambda}d(p \sin(\phi_{il}) \sin(\theta_{il}) + q \cos(\theta_{il}))}, \\ &\quad \dots, e^{j\frac{2\pi}{\lambda}d((\sqrt{N_{rx}}-1) \sin(\phi_{il}) \sin(\theta_{il}) + (\sqrt{N_{rx}}-1) \cos(\theta_{il}))}] \end{aligned} \quad (3)$$

where  $\lambda$  represents the wavelength of the signal, and  $d$  denotes the distance between antenna elements.  $p$  and  $q$  are the antenna indices in the 2D plane.

At the receiver side, Rx uses the frequency-selective baseband combiner  $\mathbf{W}_{BB}[k] \in \mathbb{C}^{N_{RF}^{rx} \times N_s}$  and the frequency-flat analog domain RF combiner  $\mathbf{W}_{RF} \in \mathbb{C}^{N_{rx} \times N_{RF}^{rx}}$  to detect and measure the transmitted signal through the wideband channel  $\mathbf{H}[k]$ . Therefore, the received signal vector at the same  $k^{th}$  subcarrier can be written as

$$\mathbf{y}[k] = \sqrt{\rho} \mathbf{W}_{BB}^H[k] \mathbf{W}_{RF}^H \mathbf{H}[k] \mathbf{F}_{RF} \mathbf{F}_{BB}[k] \mathbf{s}[k] + \mathbf{W}_{BB}^H[k] \mathbf{W}_{RF}^H \mathbf{n}[k] \quad (4)$$

where  $\mathbf{n}[k] \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_{N_s})$  is the additive white Gaussian noise vector and  $\rho$  indicates the average received SNR. Similar to the RF precoder, the RF combining matrix is implemented using phase shifters with constant modulus constraint such that  $([\mathbf{W}_{RF}]_{:,i}[\mathbf{W}_{RF}]_{:,i}^H)_{\ell,\ell} = N_{rx}^{-1}$ .

## III. HYBRID BEAMFORMING AND COMBINING DESIGN

Firstly in this work, we will focus on the hybrid precoder design, and a similar manner can be applied to design the

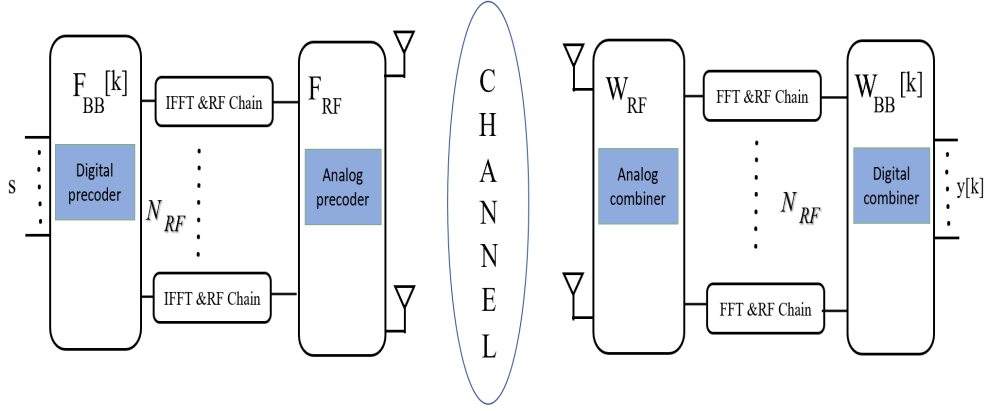


Fig. 1. MmWave multi-user system with hybrid precoding and hybrid combining.

hybrid combiner. For a hybrid architecture, the digital precoder must be designed jointly with the analog precoder. As known, the analog precoder is commonly used for all the subcarriers in the mmWave MIMO-OFDM systems. Therefore, the problem formulation to design the analog precoder and all digital precoders for  $K$  subcarriers is given as

$$\begin{aligned} \{\mathbf{F}_{BB}^{opt}[k], \mathbf{F}_{RF}^{opt}\} &= \arg \min_{\mathbf{F}_{BB}[k], \mathbf{F}_{RF}} \sum_{k=0}^{K-1} \|\mathbf{F}_{opt}[k] - \mathbf{F}_{RF}\mathbf{F}_{BB}[k]\|_F \\ \text{subject to } &\begin{cases} \|\mathbf{F}_{RF}\mathbf{F}_{BB}[k]\|_F^2 = N_s \\ ([\mathbf{F}_{RF}]_{:,i}[\mathbf{F}_{RF}]_{:,i}^H)_{\ell,\ell} = N_{tx}^{-1} \end{cases} \end{aligned} \quad (5)$$

As the summation is linear operator, the hybrid precoding of each subcarrier can be designed instead of the summation formation as follows

$$\begin{aligned} \{\mathbf{F}_{BB}^{opt}[k], \mathbf{F}_{RF}^{opt}\} &= \arg \min_{\mathbf{F}_{BB}[k], \mathbf{F}_{RF}} \|\mathbf{F}_{opt}[k] - \mathbf{F}_{RF}\mathbf{F}_{BB}[k]\|_F \\ \text{subject to } &\begin{cases} \|\mathbf{F}_{RF}\mathbf{F}_{BB}[k]\|_F^2 = N_s \\ ([\mathbf{F}_{RF}]_{:,i}[\mathbf{F}_{RF}]_{:,i}^H)_{\ell,\ell} = N_{tx}^{-1} \end{cases} \end{aligned} \quad (6)$$

where  $\mathbf{F}_{opt}[k]$  are the unconstrained optimal precoders which are found from the factorization of the  $k^{th}$  subcarrier channel  $\mathbf{H}[k]$ . In (6), the first constraint is the total transmit power to ensure the efficiency of the communication, especially in the hybrid architecture. The second constraint is hardware practical constraints of fully connected structures applied when analog precoders are implemented using phase shifters. As mentioned above, we adopt the GMD decomposition as a unitary decomposition to derive unconstrained optimal precoders and combiners from each subcarrier channel matrix  $\mathbf{H}[k]$  in order to improve the capacity performance of the mmWave MIMO OFDM system. In this work, we assume that each  $k^{th}$   $\mathbf{H}[k]$  is available at both the Tx and Rx. Further,  $\mathbf{H}[k]$  is well-conditioned to transmit  $N_s$  data streams such that  $\text{rank}(\mathbf{H}[k]) \geq N_s$ . At each  $k^{th}$  subcarrier, we write the GMD decomposition of  $\mathbf{H}[k]$  as follow

$$\mathbf{H}[k] = \mathbf{Q}_1[k]\mathbf{R}_1[k]\mathbf{P}_1^H[k] \quad (7)$$

where  $\mathbf{Q}_1 \in \mathbb{C}^{N_{rx} \times N_s}$  and  $\mathbf{P}_1 \in \mathbb{C}^{N_{tx} \times N_s}$  are semi-unitary matrices containing the left orthonormal columns of unitary

matrices  $\mathbf{Q} \in \mathbb{C}^{N_{rx} \times N_{rx}}$  and  $\mathbf{P} \in \mathbb{C}^{N_{tx} \times N_{tx}}$ .  $\mathbf{R}_1 \in \mathbb{R}^{N_s \times N_s}$  is a real upper triangular matrix with diagonal elements all equal to the geometric mean of the positive singular values [19]

$$r_{ii} = \bar{\sigma} = \left( \prod_{\sigma_j > 0} \sigma_j \right)^{\frac{1}{N_s}}, \quad 1 \leq i \leq N_s \quad (8)$$

where  $\sigma_j$  are the singular values of  $\mathbf{H}[k]$ , whereas  $\bar{\sigma}$  is the geometric mean of the positive singular values. From the unconstrained GMD-based beamforming scheme, we can exploit  $\mathbf{P}_1$  as unconstrained optimal precoders  $\mathbf{F}_{opt}$  and  $\mathbf{Q}_1$  as the unconstrained optimal combiners  $\mathbf{W}_{opt}$ . Therefore, the joint hybrid precoder design problem in (6) can be rewritten as

$$\begin{aligned} \{\mathbf{F}_{BB}^{opt}[k], \mathbf{F}_{RF}^{opt}\} &= \arg \min_{\mathbf{F}_{BB}[k], \mathbf{F}_{RF}} \|\mathbf{P}_1[k] - \mathbf{F}_{RF}\mathbf{F}_{BB}[k]\|_F \\ \text{subject to } &\begin{cases} \|\mathbf{F}_{RF}\mathbf{F}_{BB}[k]\|_F^2 = N_s \\ ([\mathbf{F}_{RF}]_{:,i}[\mathbf{F}_{RF}]_{:,i}^H)_{\ell,\ell} = N_{tx}^{-1} \end{cases} \end{aligned} \quad (9)$$

Although unconstrained optimal precoders are derived, the optimization of (9) is a nonconvex problem. As solution, the alternating minimization algorithm based manifold optimization technique decouples the global minimization problem (9) into two optimization problems and treats each problem as a matrix factorization problem. As the architecture structure of the analog part in a hybrid system is known, we can exploit this analog structure as prior information to generate the analog precoders according to a suitable mathematical formula. Then, each  $(i, j)$  element of the analog precoder matrix can be written as

$$[\mathbf{F}_{RF}]_{(i,j)} = \frac{1}{\sqrt{N_{tx}}} e^{j\varphi_{i,j}} \quad (10)$$

where  $\varphi_{i,j} \in [0, 2\pi]$  is the phase of  $i^{th}$  analog precoder column, such that,  $\varphi_{i,j}$  can be generated with uniform random distribution. For designing analog and digital precoders, we adopt the MO-AltMin algorithm to solve the matrix factorization problem. We fix the analog precoder  $\mathbf{F}_{RF}$  and neglect the constraints of problem (9) to design the digital

precoders  $\mathbf{F}_{BB}[k]$ . So, the problem designing of  $\mathbf{F}_{BB}[k]$  will be expressed as

$$\{\mathbf{F}_{BB}^{opt}[k]\} = \arg \min_{\mathbf{F}_{BB}[k]} \|\mathbf{P}_1[k] - \mathbf{F}_{RF}\mathbf{F}_{BB}[k]\|_F \quad (11)$$

by using the closed-form solution for (11) [5],  $\mathbf{F}_{BB}[k]$  can be obtained as

$$\hat{\mathbf{F}}_{BB}[k] = (\mathbf{F}_{RF}^H \mathbf{F}_{RF})^{-1} \mathbf{F}_{RF}^H \mathbf{P}_1[k] \quad (12)$$

According to the alternating scheme, and since the  $\mathbf{F}_{RF}$  is commonly applied to all the subcarriers, we fix each  $k^{th}$  subcarrier digital baseband precoder for designing the analog precoder under the following optimization problem

$$\{\mathbf{F}_{RF}^{opt}\} = \arg \min_{\mathbf{F}_{RF}[k], \mathbf{F}_{RF}} \sum_{k=0}^{K-1} \|\mathbf{P}_1[k] - \mathbf{F}_{RF}\hat{\mathbf{F}}_{BB}[k]\|_F \quad (13)$$

$$\text{subject to } ([\mathbf{F}_{RF}]_{:,i} [\mathbf{F}_{RF}]_{:,i}^H)_{\ell,\ell} = N_{tx}^{-1}$$

$$\text{for } k \in \{0, \dots, K-1\}$$

the manifold optimization with iterative method are employed to solve the problem (13) by using projection into Riemannian manifolds, where the conjugate gradient algorithm in Euclidean spaces could find the approximately optimal design of the analog precoder  $\mathbf{F}_{RF}$ . Algorithm 1 summarizes our proposed method to design the hybrid precoding using GMD decomposition for mmWave MIMO-OFDM systems.

**Algorithm 1** Alternating Hybrid precoding design algorithm using GMD decomposition

**Input:**

- 1)  $\mathbf{H}[k]$ ;  $[\mathbf{Q}[k], \mathbf{R}[k], \mathbf{P}[k]] = \text{GMD}(\mathbf{H}[k])$  by using Algorithm in [19];
- 2) Construct semi-unitary matrices  $\mathbf{Q}_1[k]$  and  $\mathbf{P}_1[k]$ ;
- 3) generate  $\mathbf{F}_{RF}^{(0)}$  using (10) with uniform random phases;
- 4) Initialize the subcarrier number  $k = 0$ ;

**Repeat**

- 5) Fix  $\mathbf{F}_{RF}^{(k)}$ 
  - for**  $k \leq K$  **do**
  - $\hat{\mathbf{F}}_{BB}[k] = (\mathbf{F}_{RF}^H \mathbf{F}_{RF})^{-1} \mathbf{F}_{RF}^H \mathbf{P}_1[k]$
  - end for**
- 6) Fix all subcarrier baseband precoders
  - Calculate  $\mathbf{F}_{RF}^{(k+1)}$  using Conjugate Gradient Algorithm
- 7)  $k \leftarrow k + 1$

**until** a stopping criterion triggers

- 8) Normalize each subcarrier baseband as:

$$\tilde{\mathbf{F}}_{BB}[k] = \sqrt{N_s} \frac{\hat{\mathbf{F}}_{BB}[k]}{\|\mathbf{F}_{RF}\hat{\mathbf{F}}_{BB}[k]\|_F}$$

#### IV. SIMULATION RESULTS

In this section, we present numerical results to evaluate the performance of the proposed method used for designing the hybrid precoding/combining in mmWave MIMO-OFDM systems. For simulations, we assume that  $N_{tx}$  and  $N_{rx}$  are equipped with  $N_{tx} = 144$  and  $N_{rx} = 36$  antennas and with the same number of RF Chain ( $N_{RF}^{tx} = N_{RF}^{rx} = 5$ ) at both  $T_x$

and  $R_x$  to exchange multi-stream data with  $N_s = 4$ . Moreover, element antennas arrays are arranged in UPA configurations with  $d = \frac{\lambda}{2}$  antenna spacing distance. The mmWave channel is generated according to the model in (2) with  $N_{cl} = 5$  clusters,  $N_{ray} = 10$  rays and the average power of each cluster is  $\sigma_{\alpha,i}^2 = 1$ . For The azimuth and elevation angles of departure and arrival, we assume that AoDs and AoAs follow the Laplacian distribution with uniformly distributed mean angles and angular spread of 10 degrees. The number of total subcarriers  $K$  is equal to 128. As mentioned above, we use the averaged spectral efficiency as a metric to evaluate the performance with the obtained hybrid precoder/combiner design. If transmitted symbols over the mmWave channel follow a Gaussian distribution, the achieved spectral efficiency can be rewritten as [15][20]

$$\mathbf{R} = \log_2 \left( \mathbf{I}_{N_s} + \frac{\rho}{N_s} \mathbf{R}_n^{-1} \mathbf{W}_{BB}^H[k] \mathbf{W}_{RF}^H \mathbf{H}[k] \mathbf{F}_{RF} \mathbf{F}_{BB}[k] \times \mathbf{F}_{BB}^H[k] \mathbf{F}_{RF}^H \mathbf{H}^H[k] \mathbf{W}_{RF} \mathbf{W}_{BB}[k] \right) \quad (14)$$

where  $\mathbf{R}_n = \sigma_n^2 \mathbf{W}_{BB}^H[k] \mathbf{W}_{RF}^H \mathbf{W}_{RF} \mathbf{W}_{BB}[k]$  is the noise covariance matrix after combining.

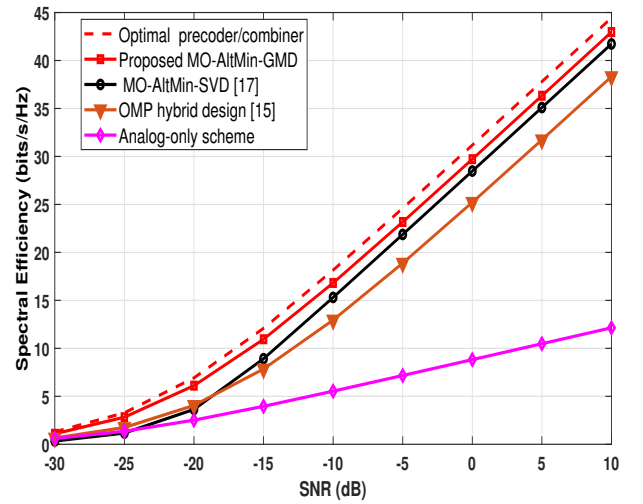


Fig. 2. Spectral efficiency v.s. SNR, achieved by different precoding/combining methods.

Fig.2 illustrates the averaged spectral efficiency performance of the fully-connected structure versus SNR over 1000 channel realizations in hybrid mmWave MIMO-OFDM system. From this figure, we can see that the spectral efficiency performance of the proposed hybrid precoder/combiner design achieves near-optimal performance with a small gap to the optimal fully digital precoder/combiner results and with a similar trend over the all SNR range. Although the MO-AltMin algorithm [17] gives higher spectral efficiency performance than the OMP algorithm [15] and analog-only scheme results. It suffers from different SNRs for each sub-channel of mmWave MIMO OFDM system, especially at low SNR range.

Fig.3 compares the spectral efficiency performance of different hybrid precoding/combining schemes versus RF chain

numbers under the condition that  $N_s = 4$  and  $SNR = 5$ . It can be seen clearly that the performance obtained by the proposed full-connected hybrid precoding/combining method is very close to the optimal full-digital precoding/combining

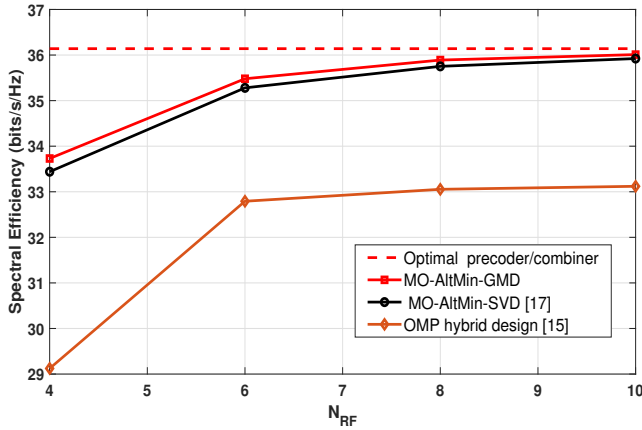


Fig. 3. The spectral efficiency performance v.s. number of RF chains under the condition that  $N_s = 4$ , and  $SNR = 5$ .

In theory, to obtain a near optimal full-digital precoding/combining performance, it is sufficient that the number of RF chains should be greater than or equal to twice of the number of data streams [2]. Although increasing the RF chain numbers, the performance improvement achieved by MO-AltMin algorithm [17] and OMP algorithm [15] is less than the performance of the proposed method.

## V. CONCLUSION

In this paper, we proposed the MO-AltMin algorithm using GMD decomposition to meet the influence of different SNRs for each subchannel, especially in wideband mmWave MIMO-OFDM systems. The use of GMD decomposition to derive the unconstrained optimal precoders/combiners achieves near-optimal performance to the optimal fully digital precoder/combiner results in term of spectral efficiency without any power allocation schemes, which allow us to avoid the use of the high complexity hybrid architecture and the limited saturation power of mmWave power amplifier (PA). Simulation results indicate that the proposed MO-AltMin-GMD method can achieve better performance than other existing methods such MO-AltMin-SVD and OMP hybrid design.

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