Neural network based generation of 1-dimensional stochastic fields with turbulent velocity statistics

Carlos Granero-Belinchón [1,2*]
[1] *Department of Mathematical and Electrical Engineering, IMT Atlantique, Lab-STICC, UMR CNRS 6285, 655 Av. du Technopôle, Plouzané, 29280, Bretagne, France.*
[2] *Odyssey, Inria/IMT Atlantique, 263 Av. Général Leclerc, Rennes, 35042, Bretagne, France.*

We define and study a fully-convolutional neural network stochastic model, NN-Turb, which generates 1-dimensional fields with turbulent velocity statistics. Thus, the generated process satisfies the Kolmogorov 2/3 law for second order structure function. It also presents negative skewness across scales (*i.e.* Kolmogorov 4/5 law) and exhibits intermittency. Furthermore, our model is never in contact with turbulent data and only needs the desired statistical behavior of the structure functions across scales for training.

I. INTRODUCTION

Turbulence is characterized by non-linear, multiscale and non-local interactions [1]. Moreover, it presents long-range dependences and intermittency [2] making it a very interesting subject of study in the field of complex systems [3] and multifractals [4].

More particularly, the generation of stochastic fields sharing the statistical behavior of turbulence has been matter of study during the last century. Thus, several stochastic models [5–9] have been proposed to recover the very known energy distribution and energy cascade in turbulence [10] as well as intermittency [11, 12]. From the first fractional Brownian motion [5, 6], only recovering the energy distribution, the modelling of turbulence evolved towards more complex fields introducing also intermittency [13] or both intermittency and energy cascade [14]. However, the generation of stochastic fields matching the statistical properties of turbulence is still challenging.

In the last decades, neural network (NN) models have given evidences of their potential to deal with non-linear complex problems [15–17]. Specifically generative NN models have been recently developed [18]. These models aim to learn the statistical distribution of some data to next create new data matching the underlying distribution [19, 20]. In the field of NN modelling of turbulence some works already appeared in the last years [21–24]. However, all these works learn from experimental or numerical turbulent data, sometimes helped by additional physic information [25, 26]. This introduces a strong dependence on databases that are not always easily available. However, a NN model is just a non-linear function Ψ_θ completely defined by the weights θ of its neurons, and so, we can formulate an optimization problem of Ψ_θ with respect to a given criterion [27, 28]. From this viewpoint we don't need to feed our model with data.

Thus, we propose to avoid learning from data and directly impose the multiscale statistical behavior described by Kolmogorov and Obukhov theories. The proposed approach is a multiscale generalization of the Generative Moment Matching Networks from Li et al. [29] and grounds on the Kolmogorov multiscale descriptions of second, third and fourth order structure functions of turbulent velocity across scales [10–12, 30]. We focus on these three functions since they are representative of the energy distribution, the energy cascade and intermittency respectively [1]. Thus, our model takes a Gaussian white noise as input and only needs the desired evolution across scales of three structure functions of turbulent velocity for learning. These functions can be defined from experimental or numerical data, but also from empirical or theoretical models.

In section II we describe the multiscale statistical behavior of turbulent velocity. Then, section III presents our NN model, that we named NN-Turb, as well as the used optimization approach. Finally, in sections IV and V we show the statistical multiscale behavior of the generated stochastic field and we give some conclusions and perspectives.

II. ISOTROPIC AND HOMOGENEOUS FULLY DEVELOPPED TURBULENCE

The Kolmogorov 1941 statistical multiscale theory prescribes the existence of three domain of scales in turbulence with different statistical behaviors: integral, inertial and dissipative domains, where the energy is respectively injected

* Correspondence: carlos.granero-belinchon@imt-atlantique.fr

in the flow, transferred across scales and dissipated [1, 10]. The integral scale L separates the integral and inertial domains, while the Kolmogorov scale η divides the inertial and dissipative ones.

The Kolmogorov 1941 theory (K41) states two statistical relationships for the turbulent velocity in the inertial domain of scales: the 2/3 and the 4/5 laws, that illustrate respectively the energy distribution and cascade across scales [1, 10]:

$$\delta_l v(x) = v(x+l) - v(x) \tag{1}$$

$$S_p(l) = \langle (\delta_l v(x))^p \rangle \tag{2}$$

$$S_2(l) \propto l^{2/3} \tag{3}$$

$$S_3(l) \propto \frac{4}{5} l \tag{4}$$

where $v(x)$ is the turbulent velocity, $\delta_l v(x)$ is the velocity increment of size l and $S_p(l)$ is the p-th order structure function. Thus, in the inertial domain the Kolmogorov 2/3 law implies that $S_2(l) \propto l^{2/3}$, and the Kolmogorov 4/5 law imposes $S_3(l) \propto l$. The 4/5 law was directly derived by Kolmogorov from the Navier-Stokes equations and must be respected by any model of turbulence [10].

Moreover, the Kolmogorov and Obukhov 1962 theory (KO62) corrected the K41 by introducing intermittency: the energy dissipation rate is inhomogeneous and should be considered locally [11, 12]. Moreover, this correction leads to the emergence of extreme values of the velocity increments both in the inertial and dissipative domains, and so, to a deformation of the probability density function (pdf) of the velocity increments when the scale decreases, from Gaussian in the integral domain to strongly non-Gaussian in the dissipative one [1, 2].

For practical purposes we define the skewness and flatness as:

$$\mathcal{S}(l) = \frac{S_3(l)}{S_2(l)^{3/2}} \tag{5}$$

$$\mathcal{F}(l) = \frac{S_4(l)}{S_2(l)^2}. \tag{6}$$

In these measures the dominant effects of the energy distribution across scales ($S_2(l)$) are compensated and so they allow us to finely study high-order statistics. On the one hand, the skewness characterizes the degree of asymmetry of the distributions of the velocity increments and from the Kolmogorov 4/5 law it is a signature of the energy cascade. On the other hand, the Flatness describes the significance of the tails of the distribution of the increments and its evolution across scales characterizes intermittency.

In this work, we will focus on four main points of the Kolmogorov-Obukhov theories that we will impose to the process generated by our NN-Turb model: a) turbulence presents three domain of scales with different statistical behaviors, b) in the inertial domain $S_2(l)$ matches the 2/3 kolmogorov law, c) in the dissipative and inertial domaines the skewness should be negative (4/5 Kolmogorov law) and d) in the integral domain the flow statistics are close to Gaussian and become non-Gaussian at small scales, *i.e.* the flatness of the velocity increments increases when the scale decreases. Thus, our NN-Turb model will generate intermittent processes with the desired energy distribution and cascade as prescribed by Kolmogorov and Obukhov.

III. TURBULENT VELOCITY NEURAL NETWORK BASED GENERATION

This section describes the proposed deep learning approach for 1-dimensional turbulent velocity fields modelling referred to as NN-Turb. We introduce our neural network model and the optimization setup.

A. NN-Turb model

We propose a fully convolutional model for the generation of 1-dimensional fields with turbulent velocity statistics:

$$\delta_{l_s} u(x) = \Psi(\mathrm{w}(x)) \tag{7}$$

where l_s is the sampling distance of the generated fields, *i.e.* the smallest available scale of analysis. Ψ is our model which takes as input a Gaussian white noise $\mathrm{w}(x)$ of size N, and produces the field corresponding to the velocity

increment $\delta_{l_s} u(x)$. The output field is also of size N. Finally, the generated turbulent velocity field $u(x)$ is defined as the cumulative sum of $\delta_{l_s} u(x)$:

$$u(x) = \sum_{i=0}^{x} \delta_{l_s} u(i) \tag{8}$$

So, our NN-Turb model performs a double operation on the input noise $\mathrm{w}(x)$. On the one hand it deformates the Gaussian pdf of $\mathrm{w}(x)$ to a pdf in agreement with turbulent velocity statistics. On the other hand, our model introduces a structure of dependencies (multi-point correlations) on the initial white noise used as input.

The operator Ψ follows a U-net architecture performing a multi-resolution processing of the input Gaussian white noise based on convolutional kernels of different sizes [31]. See A for more details on the NN-Turb architecture.

B. Optimization setup

From eq.(7) the only input of our model is a Gaussian white noise $\mathrm{w}(x)$. In addition, during training we need to introduce the desired statistics across scales of the generated process $u(x)$. Indeed, we impose the evolution across scales of $S_2(l)$, $\mathcal{S}(l)$ and $\mathcal{F}(l)$, see section II.

In this work, the reference curves of the second order structure function, skewness and flatness, that we note $S_2^r(l)$, $\mathcal{S}^r(l)$ and $\mathcal{F}^r(l)$, are obtained from statistical measures on the Modane wind tunnel dataset [32]. It consists on Eulerian longitudinal velocity measurements obtained from a grid turbulence setup. The sampling frequency of the setup was $f_s = 25kHz$, the mean velocity of the flow is $\langle v \rangle = 20.5m/s$, and the Taylor-scale based Reynolds number of the flow is $R_\lambda = 2500$. Thus the flow is considered as exhibiting fully developed turbulence. For this dataset, we use the Taylor frozen turbulence hypothesis [1] in order to interpret temporal variations as spatial ones. Thus, the sampling distance can be expressed as $l_s = \langle v \rangle / f_s$. Detailed multiscale statistical analysis of Modane turbulent velocity signals has been previously provided in [30, 32–34]. Furthermore, from previous studies the integral and Kolmogorov scales for this flow are respectively $L = 2350\, l_s$ and $\eta = 5\, l_s$ [35]. Figures 2 a), b) and c) provide respectively the evolution across scales of $\log(S_2^r)$, $\mathcal{S}^r$ and $\log(\mathcal{F}^r/3)$ in blue.

We want to point out that even if in this work the reference curves are obtained from statistical measures on real data, the used learning approach allows us to use empirical or theoretical laws to define them. Our model never sees turbulent data neither during training nor during fields generation.

Thus, we consider the optimization of our NN-Turb model according to the following four losses:

1. the square error between the reference $\log(S_2^r(l))$, $\mathcal{S}^r(l)$ and $\mathcal{F}^r(l)$, and $\log(S_2(l))$, $\mathcal{S}(l)$ and $\mathcal{F}(l)$ of the generated field $u(x)$.

$$\mathcal{L}_{S_2} = \sum_l \left(\log(S_2^r(l)) - \log(S_2(l))\right) \tag{9}$$

$$\mathcal{L}_{\mathcal{S}} = \sum_l \left(\mathcal{S}^r(l) - \mathcal{S}(l)\right) \tag{10}$$

$$\mathcal{L}_{\mathcal{F}} = \sum_l \left(\mathcal{F}^r(l) - \mathcal{F}(l)\right) \tag{11}$$

2. the Kullback-Leibler distance [36] $\mathcal{L}_{KL}$ between the centered and standardized distribution of $u(x)$ and a centered and standard Gaussian distribution.

Overall, the optimization criterion $\mathcal{L}$ is a weighted sum of these losses:

$$\mathcal{L} = \alpha \cdot (\mathcal{L}_{S_2} + \cdot \mathcal{L}_{\mathcal{S}} + \mathcal{L}_{\mathcal{F}}) + \beta \cdot \mathcal{L}_{KL} \tag{12}$$

We set empirically $\alpha = 1$ and $\beta = 0.1$.

Consequently, our optimization approach completely grounds on the Kolmogorov theories as well as on previous descriptions of turbulent velocity fields. The three square error losses want to impose the desired energy distribution and cascade across scales as well as intermittency, while the Kullback-Leibler loss is used to impose Gaussianity at large scales as desired for turbulent velocity.

Using Pytorch, our learning setup relies on Adam optimizer with a learning rate of 2e-3 for the first 100 epochs, 1e-3 for epochs between 100 and 1000 and 5e-4 for remaining epochs up to epoch 2000. The open source code is available at `https://github.com/cgranerob/NN-Turb`.

IV. RESULTS AND DISCUSSION

In this section, we study the process $u(x)$ generated by our NN-Turb model Ψ. For this purpose, we generate 256 realizations of $u(x)$, each one of size $N = 2^{15}$. To avoid border effects due to convolutions, the input noises of our model are of size $N + 8192$ samples and we only consider the N samples in the middle of the output process $u(x)$. Then, we analyse their second order structure function, skewness and flatness across scales for scales going from the dissipative domain to the integral one.

Figure 1 illustrates three realizations of $u(x)$ among the 256 generated. We observe dynamics at very different scales, from very small scales of the order of the sampling distance l_s to scales of the order of the integral scale L of the process (to facilitate visualization a blue box of width L is shown).

Figure 2 a), b) and c) show respectively $\log(S_2(l))$, $\mathcal{S}(l)$ and $\log(\mathcal{F}(l)/3)$ of $u(x)$ in function of $\log(l/L)$. The integral and Kolmogorov scales as obtained for Modane turbulent data in previous studies [35], are indicated by the vertical dashed black lines. Thus, we observe different behaviors of the studied statistics depending on the domain of scales.

In figure 2 a) we observe a plateau of $\log(S_2(l))$ for scales l larger than the integral scale L as expected for turbulent velocity. These are the more energetic scales. Then when the scale decreases, $\log(S_2(l))$ also decreases following the 2/3 law of Kolmogorov for the inertial region. In the dissipative domain $\log(S_2(l))$ decreases faster with the scale than in the inertial one, and its slope is close to 2 as described by the Batchelor model [37].

Figure 2 b) illustrates that the generated field $u(x)$ is negative skewed as expected from the Kolmogorov 4/5 law. Moreover, due to intermittency effects in the dissipative domain, the skewness decreases whith the scale for scales $l < \eta$. However, this decrease is much more steep than the one from Modane experiment.

In figure 2 c) $\log(\mathcal{F}(l)/3)$ goes from zero in the integral domain of scales to larger values when the scale decreases. This is a signature of intermittency in both the inertial and dissipative domains [11, 30, 38]. Moreover, it shows a linear behavior in the inertial domain with slope -0.1 (red dashed line). This behavior is representative of homogeneous and isotropic turbulence [30]. Furthermore, intermittency is stronger in the dissipative domain and so the increase of flatness becomes stepper [39].

Our stochastic field recovers the correct behavior of $S_2(l)$ in all the domains of scales. In addition, it recovers the good behavior of skewness and flatness in the integral and inertial domains. However, in the dissipative domain the skewness decrease is to steep and the flatness increase is not enough compared to Modane statistics. This illustrates the complexity of correctly recovering high-order statistics in this region.

Finally figure 3 shows the logarithm of the pdf of the centered and standardized increments ($\delta'_l u(x)$) of the generated field for different scales l. Scales from the dissipative domain to the integral one are considered. We observe an evolution from non-Gaussian pdfs at small scales: long-tailed and assymetric, to close to Gaussian when approaching the integral domain (centered and standardized Gaussian pdf is illustrated in black dashed line). So, the generated field $u(x)$ presents intermittency and becomes Gaussian at large scales. However, this evolution of the pdfs across scales is not exactly the one expected for a turbulent field [30]: the extreme values at small scale remain of the same order than the extreme values at large scale.

V. CONCLUSIONS

We propose a fully-convolutional NN model, NN-Turb, to generate 1-dimensional fields with turbulent statistics. Our stochastic model takes as input a Gaussian white noise and perform a double operation on it: 1) it introduces the desired structure of dependencies and 2) it deformates the Gaussian pdf of the input to a long-tailed and skewed one. Very importantly, our model only needs the aimed evolution across scales of the second order structure function, skewness and flatness for learning, and so, it does not require turbulent data.

The generated 1-dimensional field $u(x)$ correctly models the energy distribution, energy cascade and intermittency of turbulence while remaining close to Gaussian at large scales. Thus, it recovers the 2/3 and 4/5 laws of Kolmogorov as well as the flatness behavior in the inertial domain described in [30, 39]. However, we also illustrated that the pdfs of the increments of the generated fields do not necessarily match the expected behavior of turbulent ones. So, our stochastic field can reproduce the statistical behavior of the second, third and fourth order structure functions of turbulent velocity without matching the exact pdf deformation across scales.

Three main future perspectives are considered. First, the application of the proposed learning approach for generating 2D images of homogeneous and isotropic turbulent velocity. Second, the empirical definition of the structure functions $S_p(l)$ according to a limited number of parameters c_m. This will allow us to completely avoid the use of experimental data. More interestingly this will allow us to introduce these parameters c_m as inputs of our model, and so, to generate different types of processes with diverse multifractal properties. Thus, we aim to generalize this NN optimization approach, which does not need data, to other kind of non-linear physical systems. Finally, the definition of a learning setup in which we don't impose the evolution across scales of some structure functions but the evolution

of the pdfs of the increments directly. Once again, we will look for setups on which the desired pdfs could be defined from data or from theoretical models.

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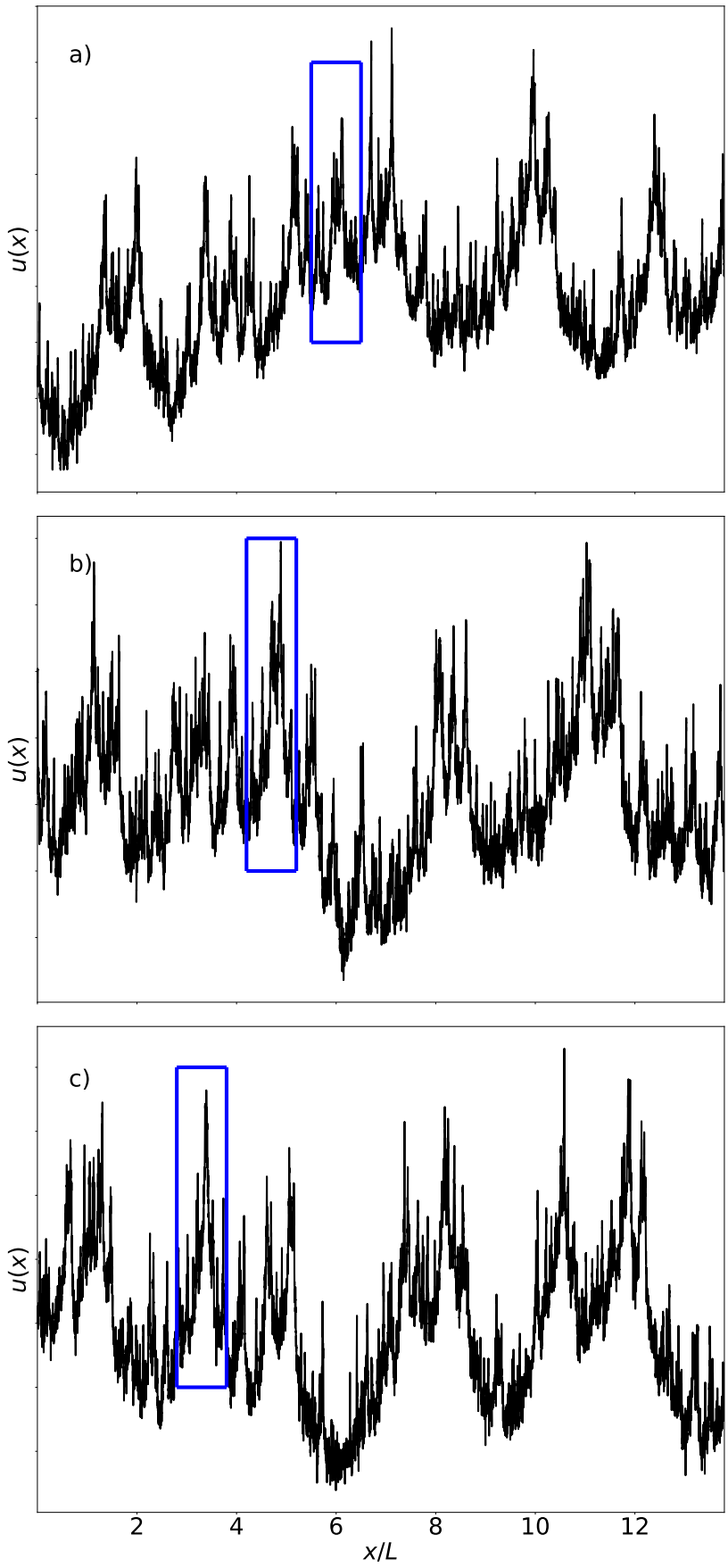


FIG. 1. Illustration of three realizations of process $u(x)$ generated with NN-Turb in function of the spatial variable x/L. The blue box corresponds to the length of an integral scale L.

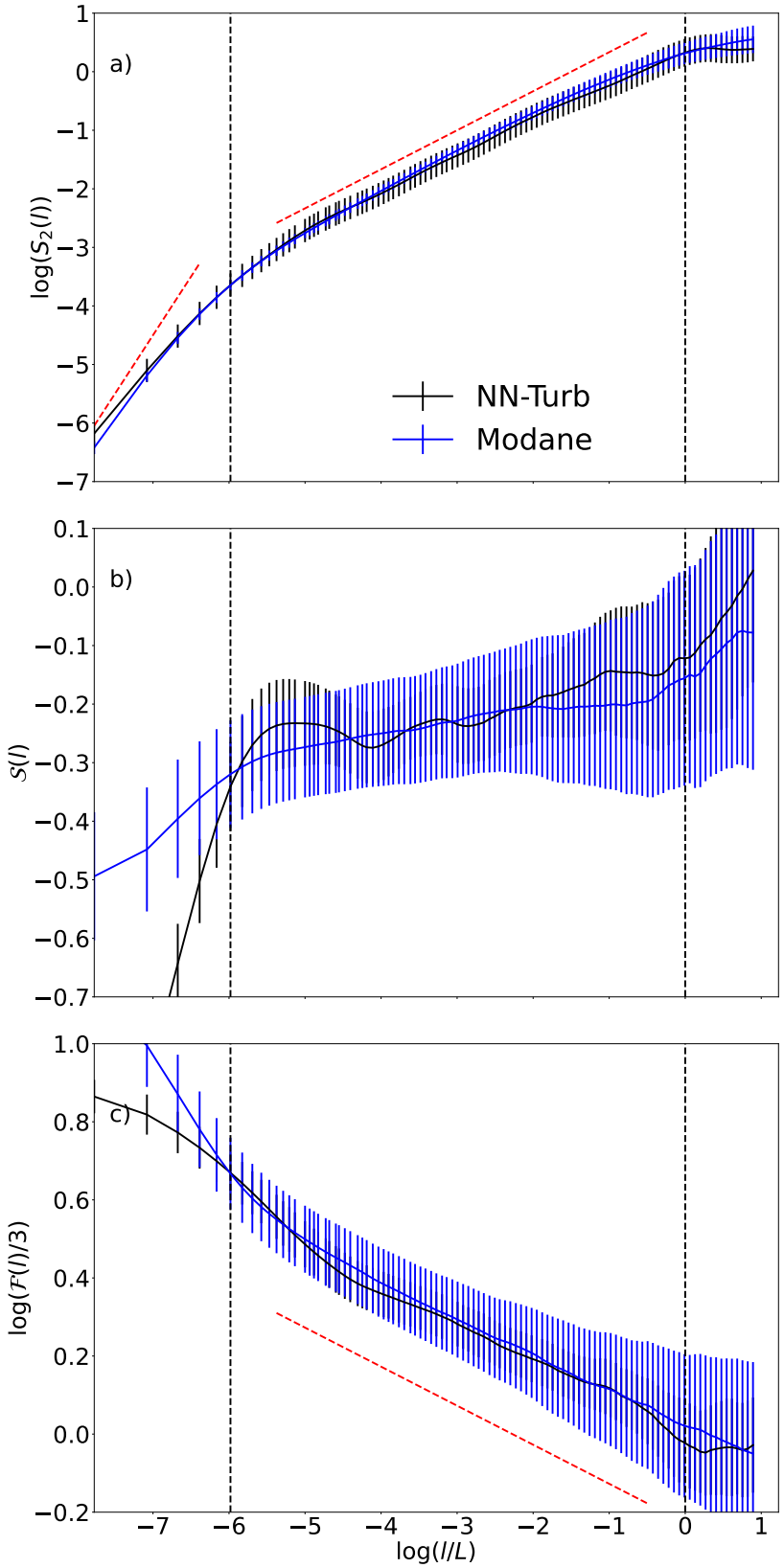


FIG. 2. a) Logarithm of the second order structure function $\log(S_2(l))$, b) skewness $\mathcal{S}(l)$ and c) logarithm of the flatness $\log(\mathcal{F}(l)/3)$ in function of the logarithm of the scale of analysis $\log(l/L)$ for the NN-Turb generated fields (black) and Modane (blue). Curves represent the mean value and errorbars the standard deviations calculated on 256 realizations. Red dashed lines in a) have a slope 2 in the dissipative domain and 2/3 in the inertial one describing respectively the behavior of the Batchelor model [37] and the 2/3 Kolmogorov law. Red dashed line in c) has a slope -0.1 previously described for the $\log(\mathcal{F}(l)/3)$ in the inertial domain [30]. The vertical black dashed lines correspond to the Kolmogorov and integral scales, η and L of Modane.

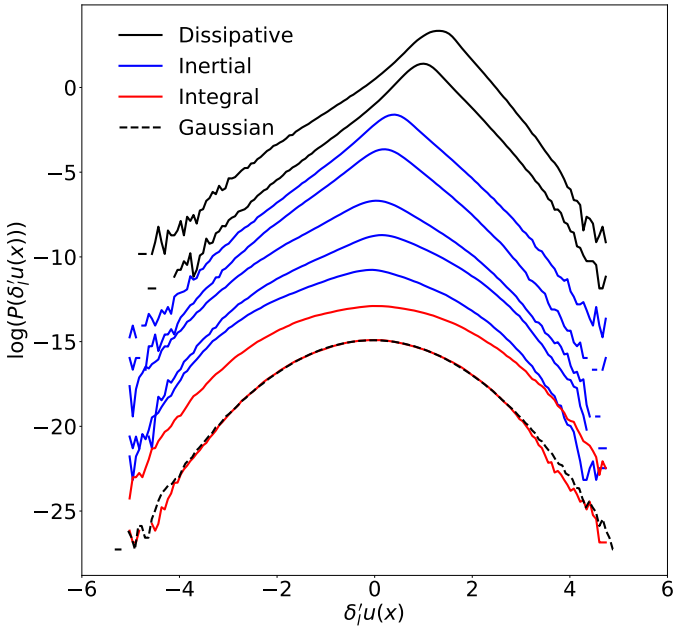


FIG. 3. Logarithm of the probability density function of the centered and standardized increments of the generated fields $\log(P(\delta'_l u(x)))$ in function of the values of $\delta'_l u(x)$. The illustrated increments are those with $l = [2, 4, 8, 16, 64, 256, 1024, 4096, 10000]\, l_s$. The integral scale of the flow is $L = 2350\, l_s$. The black dashed line correponds to the logarithm of the probability density function of a centered and standardized Gaussian distribution.

Appendix A: NN-Turb architecture

Figure 4 illustrates the architecture of NN-Turb. It is a fully convolutional U-net architecture grounding on multi-scale decomposition to modify the Gaussian white noise used as input. Thus, it is composed of an encoder, and a decoder that are connected by a convolutional bridge.

The encoder blocks are the combination of 1D convolution layer with batch normalization, non-linear ReLU activation function and average pooling. The decoder blocks are the combination of 1D convolution transpose layer with batch normalization, non-linear ReLU activation and upsampling layer. A bridge with a 1D convolution layer with batch normalization and non-linear ReLU activation followed by a 1D convolution transpose layer with batch normalization and non-linear ReLU activation is used to connect the encoder and the decoder of the U-net. The number of channels evolves as follows: $1 \to 16 \to 32 \to 64 \to 128 \to 256 \to 128 \to 64 \to 32 \to 16 \to 1$) and the kernel sizes are $[1, 2, 4, 8, 16, 32, 64]$. Furthermore, we introduced additive long-skip connections between the encoder and the decoder layers [31].

We want to point out that in both the encoder and the decoder the kernel sizes of the different layers increase exponentially in order to rapidly and completely sample the dissipative, inertial and integral domains of turbulence. This is specially important to recover the expected multi-scale behavior of each domain. Furthermore, the average pooling and upscaling layers also facilitate to process the whole domain of scales of interest without increasing dramatically the computational cost of learning.

The additive long-skip connections have been introduced to facilitate the learning [31]. Moreover, the longest skip connection, the one connecting the second and next-to-last layers, appeared as crutial to impose Gaussianity at large scales.

Furthermore, we decided to generate the smallest available velocity increment $\delta_{l_s} u(x)$ instead of directly the turbulent velocity $u(x)$ since the long-range dependences of the velocity could complicate the learning process. The range of dependences of the velocity increments are shorter.

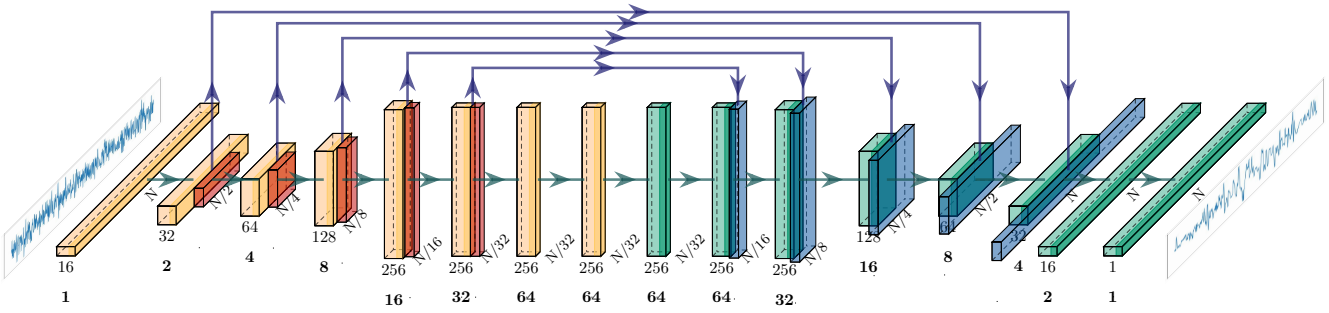


FIG. 4. Fully convolutional U-net architecture of NN-Turb model. The size of the input Gaussian white noise is N. Yellow blocks correspond to convolutional layers followed by batch normalization and ReLU activation function. Red layers correspond to average pooling of factor 2 in the spatial dimension. Green blocks correspond to convolutional transpose layers followed by batch normalization and ReLU activation. Blue layers correspond to upscaling of factor 2 in the spatial dimension. The kernel size of the convolutional layers is indicated in bold. The number of channels is also indicated for each block. Blue arrows indicate additive long-skip connections between the encoder and the decoder branches of the U-net.

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