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1 Visualization of travelling waves propagating in a plate equipped 2 with 2D ABH using wide-field holographic vibrometry

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15 16 17 **Abstract**

18
19 This paper presents a method for wide-field vibrometry based on high-speed digital
20 holographic interferometry. We demonstrate the possibility of measuring transient vibrations
21 of structures at 100kHz frame rate when providing 46600 quantitative data on 380cm²
22 rectangular spot at the object surface. Investigation of traveling acoustic waves propagating in
23 alloy plate equipped with a two-dimensional acoustic black hole (ABH) is considered. Such a
24 structure leads to localized vibrations of high amplitude and constitutes a good candidate for
25 methodology testing. The wave front is generated by a short shock with duration about 50μs.
26 The time sequence of the vibration field obtained after the shock is depicted and exhibits the
27 propagation of the wave front in the plate and inside the ABH. It follows that the observation
28 of the modification of the wave propagation can be observed at very short time scale. The
29 modification of the wave front due to the gradient in elastic properties related to the ABH area
30 is also highlighted.

31 32 33 **1. Introduction**

34 The development of lightweight structures is a field of research with many issues and
35 significant impacts. At present, several main reasons lead manufacturers to move towards a
36 reduction of weight in automobile, aeronautics, railway, etc., in order to reduce their energy
37 consumption. In this context, lightweight structures build with wave traps such as acoustic
38 black holes (ABH) are serious candidates to develop non-resonant structures without adding
39 any mass. The ABH concept was first proposed by Mironov [1] and Krylov [2]. In the past
40 years, several research groups have developed research on the analysis, modelling and the
41 characterization of ABH performance. Experimental evidence of the vibration reduction
42 induced by 1D ABH (beam termination) or 2D ABH (pit of power law profile embedded in a
43 plate) have been shown in [3,4]. The main features of the 1D ABH effect can be investigated
44 from the measurement of the reflection coefficient of a beam termination [5] and from its
45 modelling based on wave expansion [6], or wavelets expansion [7]. Spiral ABH have also

46 been proposed to reduce the size of the tapered region [8]. The 2D ABH trap effect linked to
47 the capture of the ray trajectories of flexural waves and the focalization effect has been
48 investigated in [9,10] and the scattering of the 2D ABH has been modeled in [11]. The wave
49 propagation in plates with multiple ABH indentations or ABH grids have also been modelled
50 and measured in [12,13]. Taking advantage of the vibration damping induced by ABH
51 without adding mass, first propositions of transfer of the ABH concept towards industrial
52 applications have been made [14,15]. More recently, geometrical [16] and impact [17] non
53 linearity inside the ABH have been exploited to transfer energy from the low frequency range
54 (defined as being below the ABH cut-on frequency, ~500-600Hz) to high frequencies. Such
55 strategy improves the ABH performance in the low frequency domain, in which ABH is not
56 active. Such **nonlinear systems require** analysis in the time domain, in which the present
57 metrological tool can be useful. Especially, one might be interested by characterizing their
58 global behavior when submitted to transient excitation or to investigate their nonlinear
59 properties. From the point of view of measurement and metrology, the challenge is to get
60 characterization at both the time and space scale of the involved phenomena. Generally, in the
61 domains of acoustics, vibro-acoustics, vibrations of structures or flow-induced vibrations,
62 Laser Doppler Vibrometer (LDV) is the most used tool for dynamic measurements [18-21].
63 Its main drawback is that the measurement is pointwise and a collection of data points at the
64 surface of the object has to be obtained using scanning. Such scanning may be quite long,
65 typically about 4h to scan a surface with 41×41 points. In order to overcome scanning,
66 multipoint vibrometers were developed to simultaneously yield a collection of data points at
67 the surface of the inspected object [22-26]. Nevertheless, a high density of data points can be
68 obtained by digital holographic techniques. By this way, the required measurement time can
69 be reduced when simultaneously getting high number of data points. In the past, time-
70 averaging in digital holography was discussed as an efficient tool for vibration analysis [27-
71 34]. Full object movement was also demonstrated through stroboscopic recordings [35-44].
72 However, in most of these studies, the excitation of structures was performed in the stationary
73 regime. As mentioned before, studying complex structures equipped with wave traps requires
74 analysis at both their time and space scales. It follows that providing a real-time follow-up of
75 the **vibration amplitude under arbitrary excitation conditions is a challenge** for full-field
76 optical metrology.

77 During the past years, the performances of continuous wave lasers with power larger than
78 5W, and high-speed imaging sensors (frame rate up to 1MHz) have been **significantly**
79 **improved**. Merging these technologies into holographic techniques yields an adapted

80 approach for real-time and multi-point recording of transient phenomena in complex media.
81 When recording digital holograms with a high-speed sensor, recording transient phenomena at
82 both their time and space scales becomes possible. Recently [45,46], the authors demonstrated
83 the ability of digital holography as an accurate multipoint vibrometer. Identification of the
84 force distribution at the surface of a vibrating object, by solving a regularized inverse
85 problem, was discussed when exciting with monochromatic signals [47]. In this paper, we aim
86 at considering the case of transient excitations of structures and observation over a large field
87 of view, typically larger than 350cm^2 . In literature, there does not exist any experimental set-
88 up to provide the wide-field visualization of the interaction between transient waves and 2D
89 ABH. Indeed, the use of a scanning laser Doppler vibrometer is not adapted to this case
90 because it requires spatial scanning and a perfect reproduction of the impact to the structure
91 for each scan. In the case of scanning with 41×41 data points, that would require 1681
92 impacts. A perfect repeatability for such a number of impacts is really not possible. When
93 recording the wave front propagation at its time and space scale by wide-field and time-
94 resolved digital holography, the problem of the reproducibility of impact is naturally
95 bypassed.

96 However, the spatial resolution in measurement results is also a key parameter. When
97 increasing the frame rate of any high-speed sensor, the available spatial resolution has to be
98 decreased consequently. In addition, the spatial resolution in the reconstructed images from
99 holograms closely depends on the number of useful pixels at recording and on the sensor-to-
100 scene distance. So, there is contradiction between wide-field (the scene has to be far from
101 sensor) and high spatial resolution (the frame rate has to be decreased). In this paper, on-line
102 configuration for digital holography is optimized so as to make the most of the capabilities of
103 both sensor and digital holography. A negative zoom is designed in order to fulfill the
104 Shannon requirements of holographic recording, and to provide the maximum occupation of
105 the reconstructed image of the scene in the field of view. From the reconstructed images, the
106 Doppler phase can be extracted and yields data related to the transient wave front propagation.
107 Furthermore, the light efficiency is optimized by using a dedicated diffractive optical element
108 for the surface illumination, providing a maximization of photon collection at the sensor
109 plane. Digital holography is an appropriate measurement method for capturing vibrations
110 varying rapidly with space and time. Such configuration is the one of an ABH which justify
111 the application case discussed in this paper. In the case presented in this paper, one aims at
112 visualizing the transient acoustic wave front propagating in a plate equipped with a 2D ABH,
113 and excited by a $50\mu\text{s}$ short shock from an impact hammer. The interest for the ABH leads in

114 the fact that it is an efficient way to damp the structure without adding mass. This is a
115 promising approach, having high applicative potential at long term. Nevertheless, creating
116 well controlled local damping requires understanding in details its behavior. In the case of the
117 ABH, wide-field observation of the interaction with an incident wave front contributes to a
118 better understanding of what is not. Another specificity of the ABH is that it exhibits a large
119 vibration dynamics with high variability in its central area. It also follows that ABH is a good
120 candidate to test experimental measurement methods.

121 The paper is organized as follows: Section II gives the basics fundamentals of wide-field
122 vibrometry based on digital holography, Section III describes the experimental holographic
123 set-up and Section IV discusses on the complex structure of the study. In Section V
124 experimental results are provided whereas Section VI draws the conclusion of the paper.

125

126 2. Wide-field vibrometry

127 Wide-field vibrometry provides new opportunities for studying vibrations and acoustic
128 phenomena at both their time and space scales and is based on high-speed digital holographic
129 interferometry. Digital holograms are produced from the large-field illumination of the object
130 surface to be studied. Since the complex-valued optical field is recorded in any digital
131 holograms, the optical phase, and then, the optical path difference, can be retrieved and yields
132 the measurement of the displacement field at the illuminated surface. Basically, the digital
133 holograms are obtained by recording, with an image sensor organized as a matrix of pixels,
134 the coherent mixing of the diffracted optical wave from the object surface and a known
135 reference wave. If we note O the wave front from the illuminated object and R the wave front
136 from the reference wave, then the digital hologram can be expressed by Eq. (1) (*means
137 complex conjugate) [48,49]:

$$138 \quad H = |R|^2 + |O|^2 + R^*O + RO^* . \quad (1)$$

139 The reference wave is generally written as $R(x,y)=a_R \exp[2i\pi(u_0x+v_0y)]$ with $\{u_0,v_0\}$ its spatial
140 frequencies and a_R is a constant. In this paper, the specificity of high-speed holographic
141 vibrometry is that the spatial frequencies of R can be set to $\{u_0,v_0\}=\{0,0\}$, corresponding to
142 the on-axis configuration. This point is discussed in detail in [46] and the reader is invited to
143 have a look at the paper for further details. The illuminated object surface is generally at
144 distance d_0 from the recording sensor which is used without any imaging lens (arrangement
145 known as the Fresnel configuration). The object wave diffracted to the sensor plane can be
146 expressed with the Fresnel approximations by Eq. (2) [49,50] ($i=\sqrt{-1}$):

147

$$O(x, y, d_0) = -\frac{i}{\lambda d_0} \exp\left(\frac{2i\pi d_0}{\lambda}\right) \exp\left(\frac{i\pi}{\lambda d_0} (x^2 + y^2)\right) \quad (2)$$

$$\times \iint A(X, Y) \exp\left(\frac{i\pi}{\lambda d_0} (X^2 + Y^2)\right) \exp\left(-\frac{2i\pi}{\lambda d_0} (xX + yY)\right) dXdY.$$

148 The object wave front at the object plane is $A(X, Y) = A_0(X, Y) \exp[i\psi_0(X, Y)]$, λ is the wavelength
 149 of light, A_0 is related to the object reflectance and ψ_0 is the optical phase related to the object
 150 surface profile and roughness.

151 From the digitally recorded holograms, the reconstruction of the object field at any distance d_r
 152 from the recording plane is given by the discrete Fresnel transform [48]. From the hologram,
 153 the numerically reconstructed complex-valued image can be obtained from Eq. (3) [48,49]:

154

$$A_r = h_F \times FFT[H \times h_F], \quad (3)$$

155 where FFT means two-dimensional Fast Fourier Transform and h_F is the Fresnel kernel
 156 defined by Eq. (4),

157

$$h_F(x, y) = \frac{1}{\sqrt{\lambda d_r}} \exp\left(i\pi \frac{d_r}{\lambda} - i\frac{\pi}{4}\right) \exp\left[\frac{i\pi}{\lambda d_r} (x^2 + y^2)\right]. \quad (4)$$

158 From the numerical computation of Eq. (3), the amplitude and phase of the diffracted field A_r
 159 can be evaluated. When the reconstruction distance is $d_r = -d_0$ the initial object plane is
 160 recovered and the phase variation from the time sequence is related to the displacement field
 161 at the surface. When considering two consecutive time-instants in the hologram sequence, the
 162 phase variation is given by Eq. (5) and is similar to the Doppler effect, but from the point of
 163 view of the optical phase. The phase change is thus related to the displacement field \mathbf{U} rather
 164 than the velocity.

165

$$\Delta\varphi = \frac{2\pi}{\lambda} \mathbf{U} \cdot (\mathbf{K}_e - \mathbf{K}_o). \quad (5)$$

166 In Eq. (5), \mathbf{K}_e is the normalized illumination vector from the light source to the object and \mathbf{K}_o
 167 is the observation vector (also normalized) from the object to the sensor, both defined in a set
 168 of reference axis $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ attached to the object surface, with \mathbf{k} being perpendicular to the
 169 surface. In the approach described in this paper, the observation vector is parallel to \mathbf{k} and the
 170 illumination vector is quasi-oriented along $-\mathbf{k}$. Thus, the sensitivity of the phase measurement
 171 is oriented along \mathbf{k} so that the out-of-plane movement at the surface of the object, u_z , can be
 172 measured. Generally, the phase variation in Eq. (5) is obtained modulo 2π and requires phase
 173 unwrapping to yield u_z [51].

174 Thanks to the coherent mixing by heterodyning with the reference wave (Eq. (1)), the object
 175 wave O is amplified by the reference wave R , because of the term R^*O included in the
 176 recorded hologram (third term of Eq. (1)). So, a weak object wave, due to a non-cooperative
 177 target, may be balanced by a strong reference wave, if $|R|^2 \gg |O|^2$. In addition, the reference
 178 wave is directly impacting the sensor and this makes it easier to get large amount of photons
 179 for optimizing light detection. In the approach described in this paper, measurements are
 180 possible with about 40% of the full sensor dynamics and ratio $|R|^2/|O|^2$ at about 100, thus
 181 yielding suitable phase maps for visualization or metrology purposes.

182 Image reconstruction according to Eq. (3) imposes the spatial resolution in the image plane. It
 183 depends on the sensor-to-object distance, on the pixel pitch of the sensor (p_x) and on the
 184 number of pixels of the recorded hologram. If we note $\{M,N\}$ the number of pixels along
 185 respectively the vertical and horizontal direction of the sensor plane, then the spatial
 186 resolutions achieved using Eq. (4) are given by [49]:

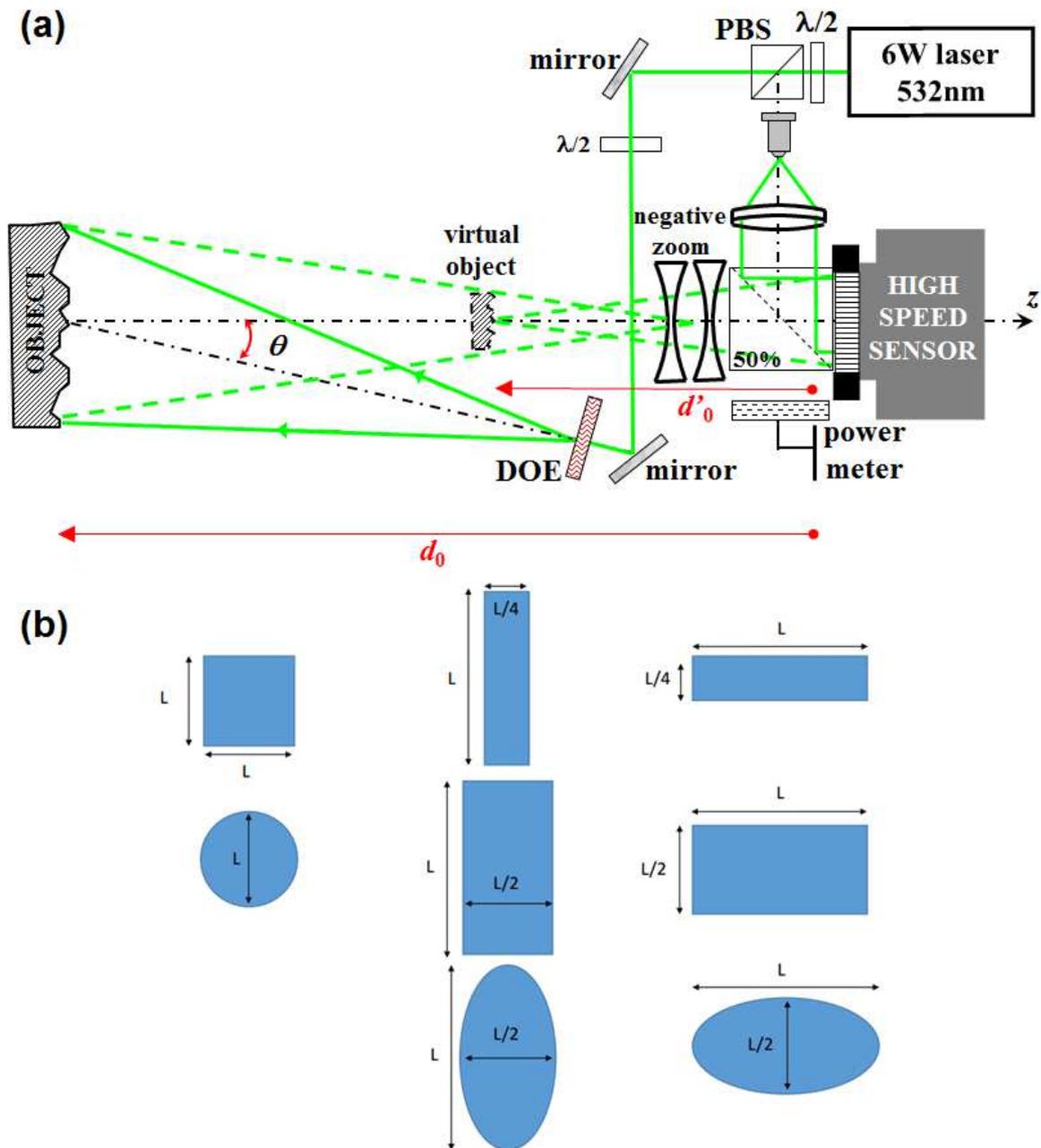
$$187 \quad \begin{cases} \rho_x = \frac{\lambda d_0}{N p_x} \\ \rho_y = \frac{\lambda d_0}{M p_x} \end{cases} \quad (6)$$

188 It follows that if the recording matrix is not square, the spatial resolution in the image plane is
 189 anisotropic. Typically, at frame rate 100kHz, the sensor of the set-up provides $N=384$ and
 190 $M=264$ pixels.

191 3. Experimental set-up

192 The experimental set-up is described in Fig. 1(a). The light is emitted from a continuous
 193 DPSS laser at $\lambda=532\text{nm}$ with maximum power at 6W. The laser is separated into a reference
 194 wave and an object wave by using a polarizing beam splitter (PBS). The half-wave plate at the
 195 output of the laser is used to adjust the power in both object and reference paths to get
 196 adequate $|R|^2/|O|^2$ ratio. This ratio is an important parameter of the experimental set-up. In [46]
 197 was demonstrated that in order to minimize the influence of measurement noise and to get
 198 contrasted phase fringe patterns, the ratio has to be adjusted at around 100. The way to adjust
 199 the ratio is as follows: the power meter in Fig. 1(a) placed near the 50% beam splitter cube is
 200 used to measure the optical power in the reference beam and in the object beam (by blocking
 201 one after the other). Using the half wave plate at the output of the laser, the ratio can be set to
 202 around 100 when splitting light in the two arms. The polarization of the object wave is then
 203 rotated 90° to be parallel with that of the reference wave, so that interferences may occur. The
 204

205 reference wave is expanded, spatially filtered using a spatial filter (microscope objective and
 206 pinhole), and collimated to produce a smooth plane reference wave impacting the sensor at
 207 normal incidence. So, the carrier spatial frequencies along the reference beam are
 208 $\{u_0, v_0\} \approx \{0, 0\}$, giving the on-line configuration [45].



209
 210 Fig. 1. (a) Experimental set-up for wide-field holographic vibrometry (PBS: polarizing beam splitter, DOE:
 211 diffractive optical element, $\lambda/2$ half-wave plate), (b) set of beam shape structures that can be produced by the
 212 DOE for illuminating the object surface

213
 214 The object wave is spatially expanded to illuminate the structure by using a dedicated DOE
 215 (Diffractive Optical Element). The DOE inserted in the illumination path is an optical component

216 that transforms the incident wave front into a desired wave front. It was designed to produce several
 217 spot shapes, with 8 subareas, each of them producing a particular laser beam shape. Figure 1(b)
 218 illustrates the diversity of shapes that can be produce with the DOE: square area, elliptical
 219 areas, narrow and large rectangular beams (vertical and horizontal). Such beam shaping
 220 increases the photometric efficiency of the set-up by avoiding wasting light with classical
 221 lenses and mirror assembly. In practice, a DOE consists of a substrate (a glass plate for example) on
 222 which a photoresist is deposited by spin-coating. The photoresist S1800 series from Micro Resist
 223 Technology was used, which is a positive photoresist. This series has different viscosities and
 224 therefore permits to put down photoresist layers with different thicknesses ranging from a few hundred
 225 nm to above 10microns. The error in the uniformity of the photoresist layer in the spin-coating process
 226 is about 20nm. Within this photoresist, micro or nano diffractive structures are engraved [50]. For
 227 application in vibroacoustics, centrosymmetric diffractive patterns are required (see Fig. 1(b)). They
 228 can be obtained through binary phase DOEs (phase shift of π between the 2 levels), the simplest to
 229 produce. The spin-coating speed should be chosen so that the thickness of the photoresist layer is
 230 $e=\lambda/2(n-1)$, where n is the refractive index of the photoresist material at the DOE working wavelength
 231 λ [52]. In this paper, the illuminating wavelength is 532nm and the photoresist thickness is close to
 232 405nm. To solve the inverse diffractive problem, diffractive structures are modelled using an Iterative
 233 Fourier-Transform Algorithm (IFTA) optimization algorithm [53,54]. Once simulated, the DOEs are
 234 fabricated in a clean room using a micro-photolithography system developed at IMT Atlantique [55].
 235 In the fabrication process, the parallel direct-write photo-plotter uses a programmable liquid crystal
 236 spatial light modulator (SLM) as a reconfigurable mask. The photoresist is then exposed to a pattern
 237 (corresponding to the modelled diffractive structures) of intense light with a wavelength of 436nm to
 238 which the photoresist is active. A reduction lens is used to image the pattern at LCD into the
 239 photoresist layer. The resolution limit of the direct-write photo-plotter is about 750nm. The pattern can
 240 be replicated along x and y axes by moving a nano-precision 2D translational stage. This fabrication
 241 process has been shown to be cost-effective and particularly adapted to DOE prototyping. After
 242 exposure, the substrate is put into a developer solution (MicropositTM 351 developer, Rohm and Haas)
 243 to etch the exposed pattern into the photoresist layer. The fabricated structures are then measured with
 244 an interferometric microscope, where the lateral dimensions and the etching depth can be verified. The
 245 quality of a DOE is assessed depending on the application, usually by diffraction efficiency,
 246 uniformity and/or mean-square-error [56,57].

247 In the set-up, the illumination angle is about $\theta=15^\circ$. From Eq. (5), the out-of-plane
 248 displacement field at the surface of the object is given by Eq. (7) [35]:

$$249 \quad u_z = \frac{\lambda}{2\pi} \frac{\Delta\varphi}{1 + \cos(\theta)}. \quad (7)$$

250 The object and reference waves are combined by the 50% beam splitter cube placed just in
 251 front of the high-speed sensor. In the optical path from the object surface to the sensor plane,
 252 a negative zoom is inserted, in front of the cube. **The negative zoom is a divergent optical**
 253 **system having a negative focal length. The image produced by such optical system is not**
 254 **located in the sensor plane, but at shorter distance from the sensor.** This negative zoom
 255 produces a smaller image of the object when reducing the object-to-sensor distance [58,59].
 256 **This provides a smaller virtual object facing the sensor at smaller distance d'_0 .** By this way,
 257 the dimensions of the virtual object are compatible with the requirement from the Shannon
 258 conditions for recording digital holograms [49]. Basically, from the hologram, the virtual
 259 image can be computed when setting $d_r = -d'_0$. And the physical plane is obtained by scaling
 260 the set of reference coordinates attached to the image plane. If we note g_{opt} the optical
 261 magnification ($0 < g_{opt} < 1$) produced by the negative zoom, then the spatial resolutions in the
 262 final image are given by

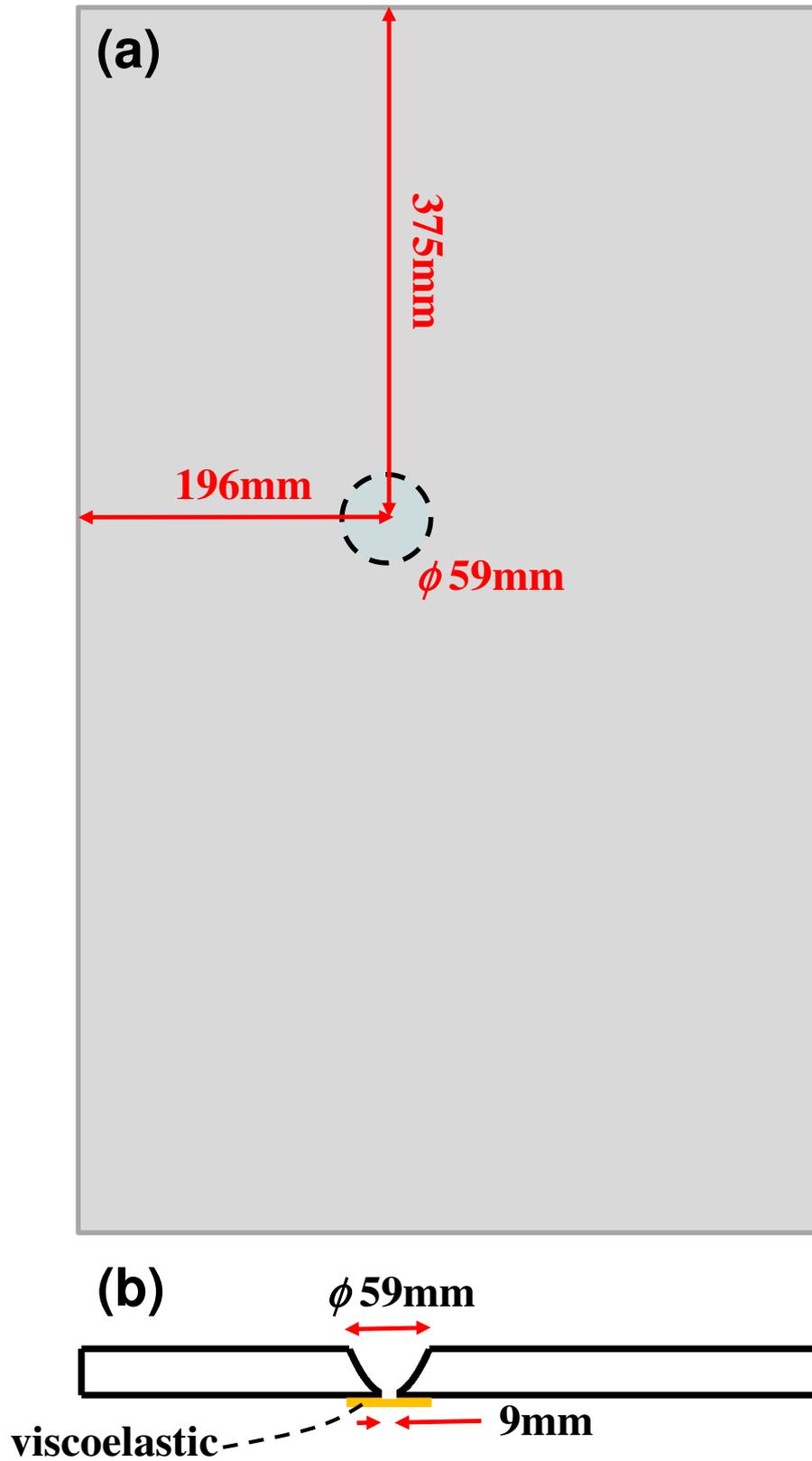
$$263 \quad \begin{cases} \rho_x = \frac{\lambda d'_0}{N p_x g_{opt}} \\ \rho_y = \frac{\lambda d'_0}{M p_x g_{opt}} \end{cases} \quad (8)$$

264 The sensor is a high-speed camera from Photron, with pixel pitch at $p_x = 20 \mu\text{m}$ and maximum
 265 spatial resolution including 1024×1024 pixels. At the full spatial resolution, the maximum
 266 frame rate is 12500Hz. When increasing the frame rate, the spatial resolution is degraded, that
 267 is 328×768 at 50kHz and 264×384 at 100kHz. The exposure time can be set from 380ns to
 268 few ms. In this paper, the exposure time was set at $1 \mu\text{s}$ and the laser power was adjusted at
 269 3W. When adjusting the negative zoom to capture holograms from a rectangular area sized
 270 $27\text{cm} \times 14\text{cm}$, about 380cm^2 , the focal length was set to -42.8mm , leading to the
 271 reconstruction distance at $d'_0 = -160\text{mm}$. In this set-up, the distance between the initial object
 272 plane and the sensor plane is about 2.45m and the optical magnification is $g_{opt} = 0.0152$. So, at
 273 100kHz, with the experimental parameters the spatial resolutions are respectively given along
 274 the x and y direction by

$$275 \quad \begin{cases} \rho_x = 792 \mu\text{m} \\ \rho_y = 1.06 \text{mm} \end{cases} \quad (9)$$

276
 277 **4. Plate equipped with a 2D ABH**

278 Wide-field holographic vibrometry is applied to the visualization of travelling waves
279 propagating in a plate equipped with a 2D ABH, when exciting using an impact hammer.
280 From Mironov [1] and Krylov [2], the ABH is known to provide an efficient vibration damper
281 for flexural waves on panels when the wave trap is made with a local variation of the
282 thickness. Generally, the truncation effect near the ABH extremity can be overcome by
283 adding a thin damping layer in its vicinity [2]. For building panels with damping properties,
284 the circular geometry of ABH was proposed and developed in [3,4]. We manufactured a 2D
285 ABH in alloy plate sized 900mm×540mm with 1.5mm thickness. The ABH zone is $\phi=59$ mm
286 in diameter and is located at 196mm from the left edge and 375mm from the top of the plate.
287 The center of the ABH is open with 9mm diameter. The plate equipped with the ABH and the
288 thickness profile of typical 2D ABH are depicted in Fig. 2.



289
 290
 291
 292

Fig. 2. (a) Scheme of the plate equipped with the 2D ABH, (b) profile of the plate equipped with the 2D ABH with details of the shape of the structure and the thin viscoelastic layer (in orange color) stuck at the ABH center

293 The principle of the ABH effect is as follows: the extremity of the structure is shaped so that
 294 the local phase velocity of flexural wave decreases progressively to 0. The consequence is that
 295 the ABH area behaves as a trap for the wave propagating through, leading to energy
 296 absorption and thus avoiding reflection phenomenon from the ABH. Figure 2(a) shows an
 297 overview of the plate equipped with the 2D ABH and its location in the plane of the plate. In
 298 the ABH area, the local thickness decreases with a power law profile as proposed by Mironov
 299 (quadratic profile) [1]. This means that the center of the ABH area is shaped so that its
 300 thickness $h(x)$ progressively decreases to 0 according to Eq. (9):

$$301 \quad h(x) = \begin{cases} e_0 (x - x_0)^2 & \text{if } |x - x_0| \leq \phi / 2 \\ e_0 & \text{if not} \end{cases} . \quad (9)$$

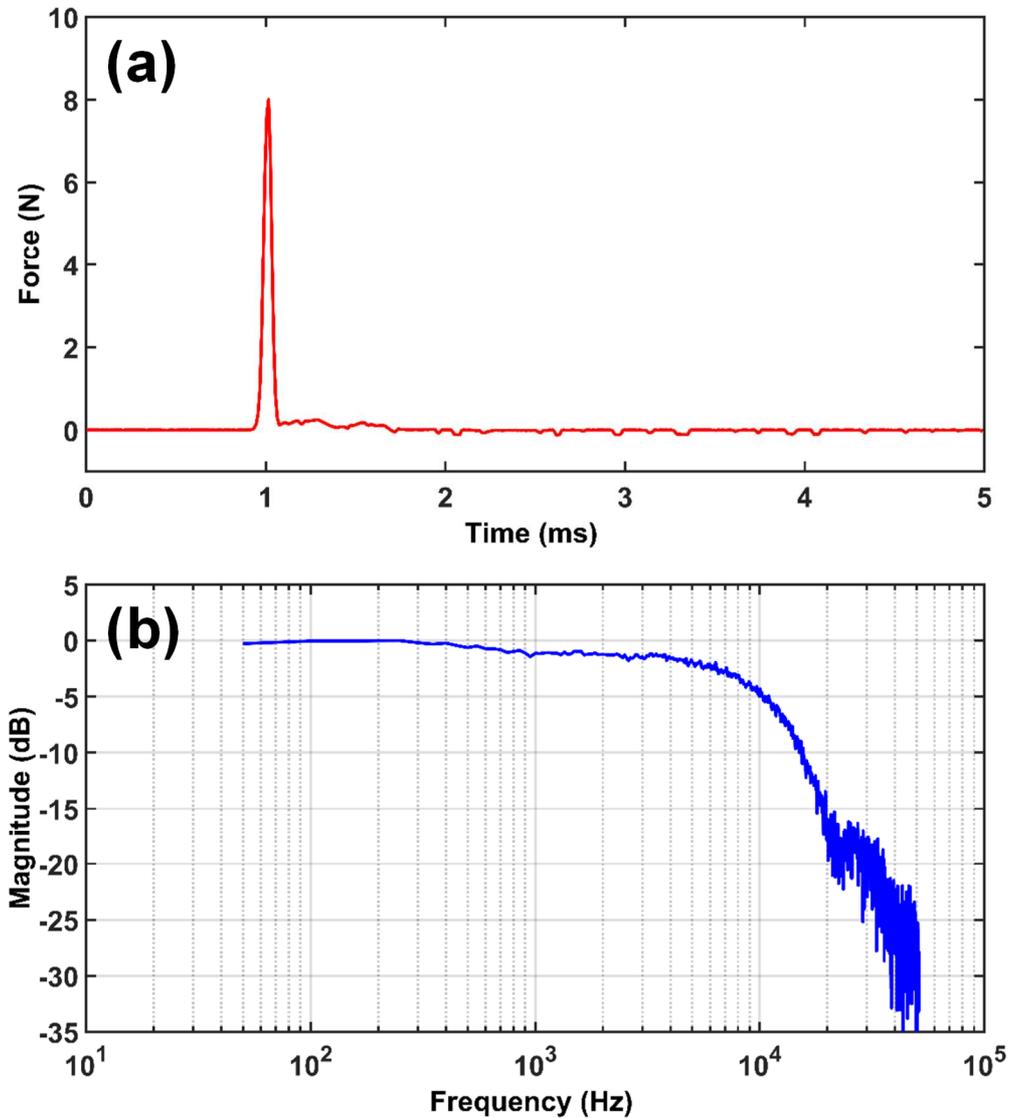
302 In Eq. (9), x_0 is the center of the ABH area and ϕ its diameter, whereas e_0 is the initial
 303 thickness of the plate. A thin viscoelastic damping layer (a viscoelastic tape having a
 304 thickness at few 20-50 μ m) has been added in the center of the ABH area. The system
 305 constituted by the ABH and the viscoelastic layer is what is usually called a ‘‘practical ABH’’:
 306 in the studied case, preliminary tests show that it is able to damp vibration from a cut-on
 307 frequency of about 500-600Hz.

308

309 **5. Experimental results**

310 In the case presented in this paper, one aims at visualizing the transient acoustic waves
 311 propagating in the structure equipped with the 2D ABH, and excited by transient signal.
 312 Therefore, the plate was excited with an impact hammer from PCB Piezotronics (ref.
 313 086E80), equipped with a force sensor, the tip being equipped with a stiff PVC tip. The
 314 hammer is used to provide shocks to the structure. The stiff tip permits to get a broad range of
 315 frequency, typically up to 10kHz. The impact of the hammer is localized at 170mm from the
 316 right edge of the plate and 292mm from its top, thus 38mm above the illuminated area. Figure
 317 3(a) shows the force signal recorded with the set-up. The maximal force is about 8N. The
 318 duration of the shock is about 50 μ s (estimated at half width). The power spectrum density of
 319 the force signal injected in the plate is provided in Fig. 3(b). As can be seen, the spectrum is
 320 broad and flat up to almost 10kHz. The force sensor is used to trig the sensor in order to
 321 record a hologram sequence. A sequence including 37600 digital holograms was recorded at
 322 the sensor frame rate of 100kHz that is duration about 0.376s, with time shift between two
 323 consecutive holograms at 10 μ s. **Considering the frequency bandwidth in Fig. 1(b), the**
 324 **Shannon conditions are fulfilled for the hologram temporal recording. Indeed, the frame rate**

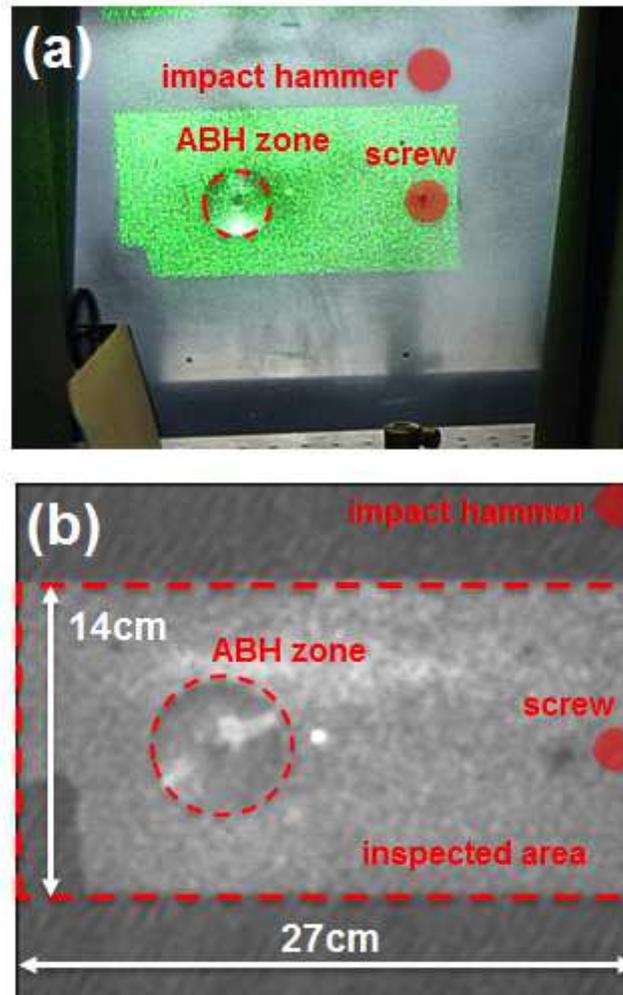
325 of the sensor is at 100kHz and the bandwidth of the shock is 20kHz. So, the temporal
326 recording provides at least 5 sampling points for the maximum frequency in the temporal
327 excitation signal.



328
329 Fig. 3. (a) Force signal injected in the plate, (b) power spectrum density of the force signal

330 The digital holograms are reconstructed using the algorithm described in Eq. (3) with
331 computation over 1024×1024 data points [49]. Figure 4(a) shows a picture of the plate
332 illuminated with the horizontal rectangular laser shape. The ABH area is indicated by a
333 dashed red line and the position of the impact hammer is marked by the red spot. Note that, in
334 the plate, there is a screw in the illuminated area, which was used to clamp a mechanical
335 shaker in previous experiments and which does not have specific role in the proposed set-up.
336 Interestingly, the influence of this small screw can be seen by the measurement method.
337 Figure 4(b) shows the amplitude image reconstructed from digital holograms. The illuminated

338 area marked with the dashed red line can be clearly observed. The ABH area, the hammer
339 position and the screw are also indicated.



340
341 Fig. 4. (a) Picture of the plate illuminated with the rectangular laser shape, the ABH area is indicated by a dashed
342 red line and the position of the impact hammer is marked by the red spot, (b) amplitude image reconstructed
343 from digital holograms, the illuminated area is marked with the dashed red line
344 With the spatial resolution evaluation from Eq. (8), the dashed red lined area in Fig. 4(b) is
345 covered by 353×132 independent data points, thus providing a collection of 46600
346 measurement points at the surface of the plate. From the complex valued data computed from
347 the reconstruction algorithm, the optical phase at each time instant can be extracted and the
348 phase difference can be obtained. Optical phase differences are obtained modulo 2π and they
349 need to be unwrapped [51] to get the scaled physical displacement field according to Eq. (7).
350 However, modulo 2π phase variations are helpful for visualizing the interaction between the
351 2D ABH and the wave front emitted from the shock with the impact hammer. Indeed, the
352 phase is wrapped in the interval $[-\pi, +\pi]$ and that permits to keep the dynamic range of display
353 constant over the set of figures, even if the wave amplitude is weak. But, in order to also

354 provide amplitude maps, the wave amplitude was calculated and displayed after unwrapping
355 the modulo 2π phase data according to Eq. (7) with $\lambda=532\text{nm}$ and $\theta=15^\circ$. In the followings,
356 Figure 5 to Fig. 9 show several sets of modulo 2π phase maps extracted from the phase
357 variation sequence after computation from the recorded digital holograms. In Fig. 5 to Fig. 9,
358 the dashed black circle indicates the ABH area. Fig. 10 to Fig. 12 show amplitude maps from
359 Fig. 6 to Fig. 8, with the dynamic range adjusted to $\pm 220\text{nm}$.

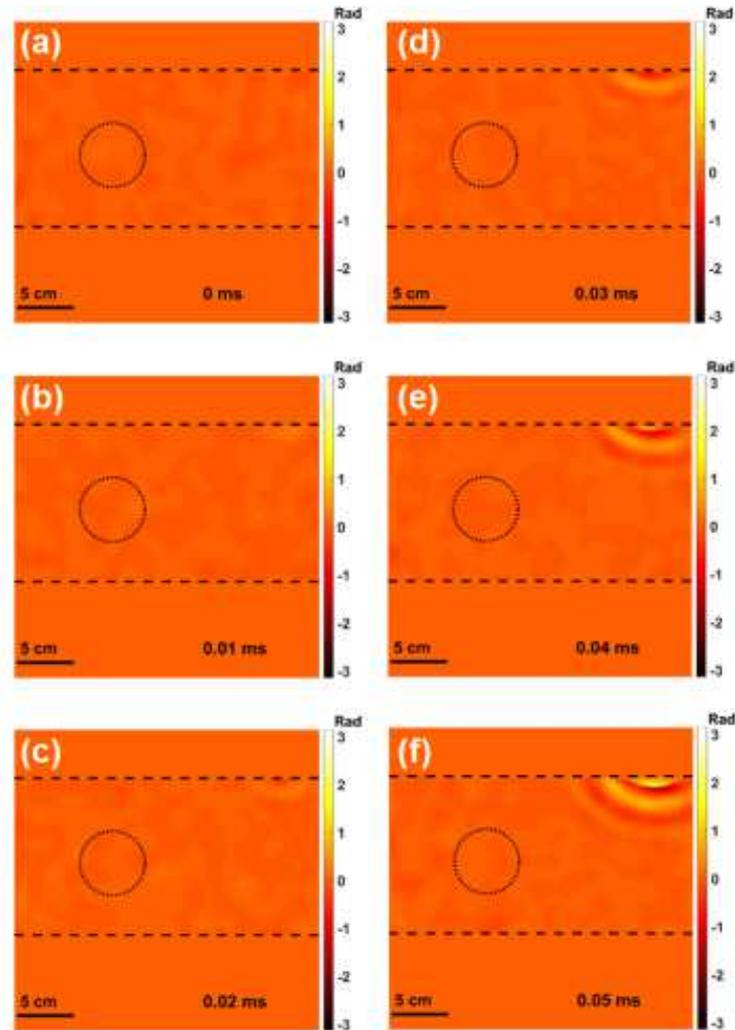
360 Figure 5(a) to Fig. 5(f) show six phase maps corresponding to the time interval [0ms; 0.05ms]
361 and showing the first instants of the wave front arrival in the region of interest illuminated by
362 the laser. The spherical wave front emitted from the impact point and propagating through the
363 plate at the early first instants can be observed. After 0.12ms the wave front reaches the screw
364 screwed to the plate. Figure 6(a) to Fig. 6(f) exhibit the interaction between the wave front
365 and the screw. One can clearly observe both phenomena: reflection and diffraction of the
366 wave by the screw. Figure 6(b) to Fig. 6(e) show the diffraction by the screw.

367 Figure 7(a) to Fig. 7(f) provides pictures of the wave front when entering in the ABH area.
368 The vanguard of the wave front impacts the area 0.19ms after the shock. Considering the
369 gradient of velocity in the ABH area, this part of the wave front has its velocity decreased.
370 The wave front is then distorted and this can be observed in the sequence of pictures in Fig. 7.
371 In Fig. 7(c), 0.21ms after the shock, the incident wave front is deformed by the velocity
372 gradient in the ABH area: the local curvature of the wave front in the ABH area changes its
373 sign. In Fig. 7(e) and Fig. 7(f), respectively 0.23ms and 0.24ms after the shock, one can
374 observe the dislocation of the wave front in the ABH zone. Note that one observes
375 concentrated phase jumps in the ABH center which indicate that the local vibration amplitude
376 increases. **These phase jumps are not spatially resolved. It follows that in Fig. 10 to Fig. 12**
377 **displaying the amplitude of the vibration, the quantitative data are not relevant in the ABH**
378 **circled area.** This is well correlated with works at LAUM demonstrating the high
379 concentration of vibration energy at the edges of the 2D ABH [12]. Note that in order to
380 observe the wave front dislocation with better spatial resolution, one would have to change
381 the experimental optical parameter in order to get a more focused imaged area, to yield
382 resolved phase jumps in the ABH center. Figure 8 shows a set of modulo 2π phase maps
383 showing the vibration field few instants after the wave front passed through the ABH area, in
384 between 0.31ms and 0.36ms after the shock. The deformation of the wave front in the lower-
385 left part of the field of view can be observed, whereas the ABH center part exhibits non
386 resolved phase jumps which indicate local high vibration amplitude. After a while, about

387 16ms after the shock, at the periphery of the ABH, a vibration field with lower amplitude is
388 observable, whereas the center still exhibits phase jumps indicating local high vibration
389 amplitude. This is highlighted in Fig. 9(a) to Fig. 9(f) which show the sequence of pictures
390 15.95ms to 16ms after the shock.

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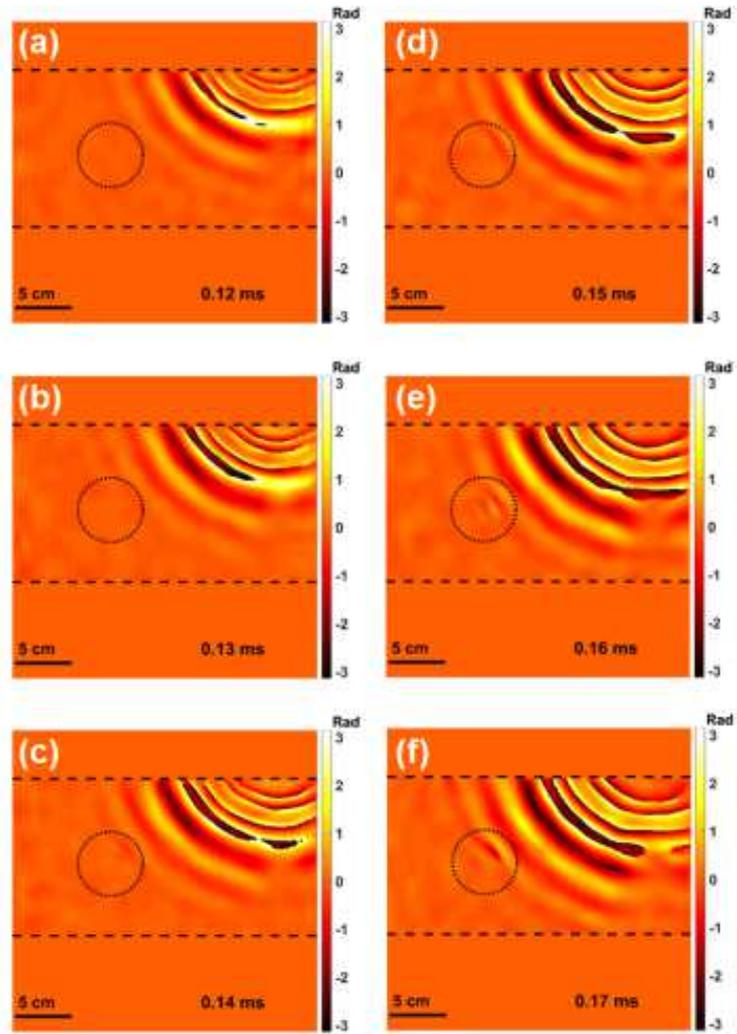


393

394 Fig. 5. Extracted modulo 2π phase maps over time interval [0ms; 0.05ms] showing the first instants of the wave
395 front arrival in the region of interest illuminated by the laser

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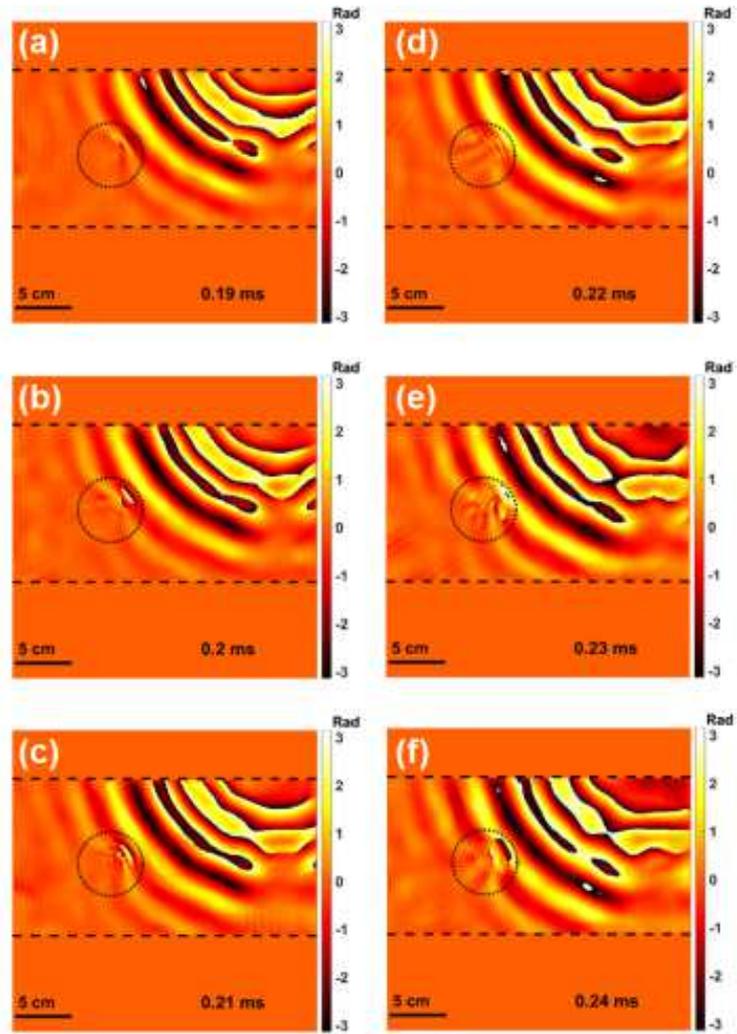
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399 Fig. 6. Extracted modulo 2π phase maps over time interval [0.12ms; 0.17ms] showing the wave front interacting
 400 with the screw inserted in the plate; the reflection of the wave front onto the screw can be visualized

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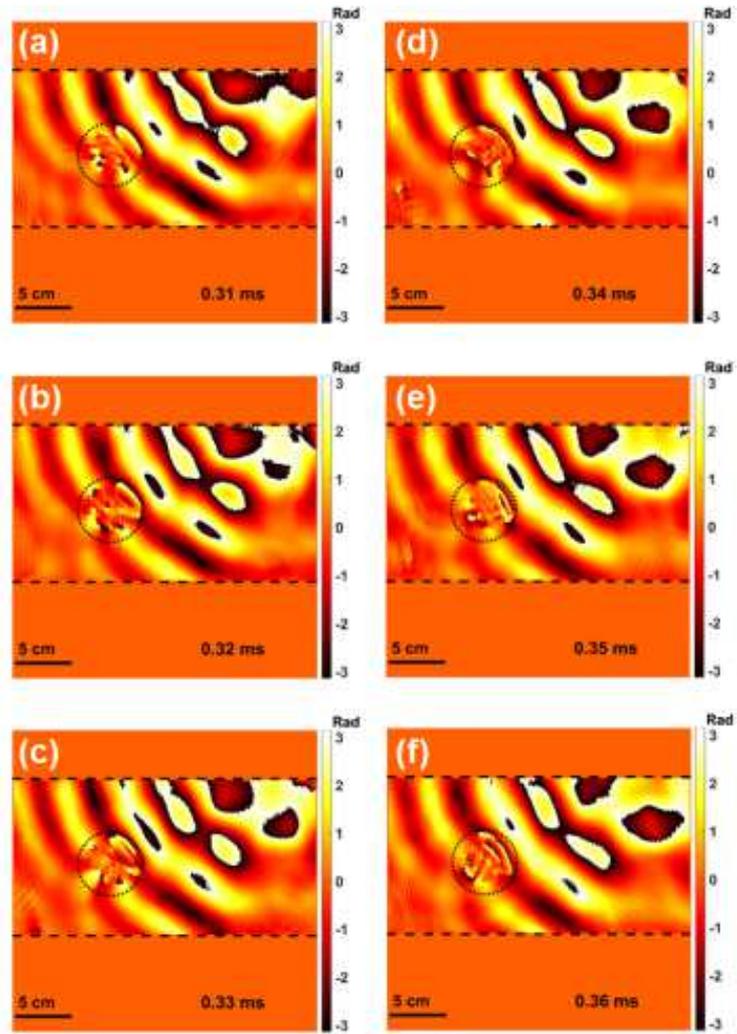
402

403 Fig. 7. Extracted modulo 2π phase maps over time interval [0.19ms; 0.24ms] showing the wave front interacting

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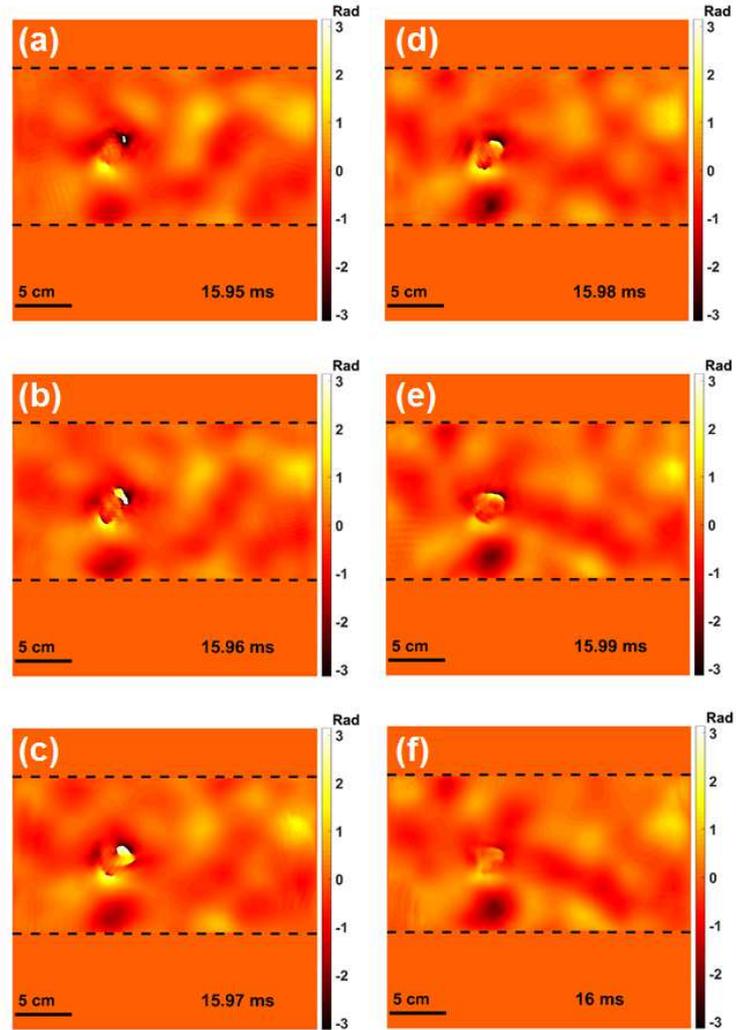
with the ABH area; the distortion of the wave front due to the velocity gradient is visualized

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Fig. 8. Extracted modulo 2π phase maps over time interval [0.31ms; 0.36ms] showing the vibration field few instants after the wave front passed through the ABH area



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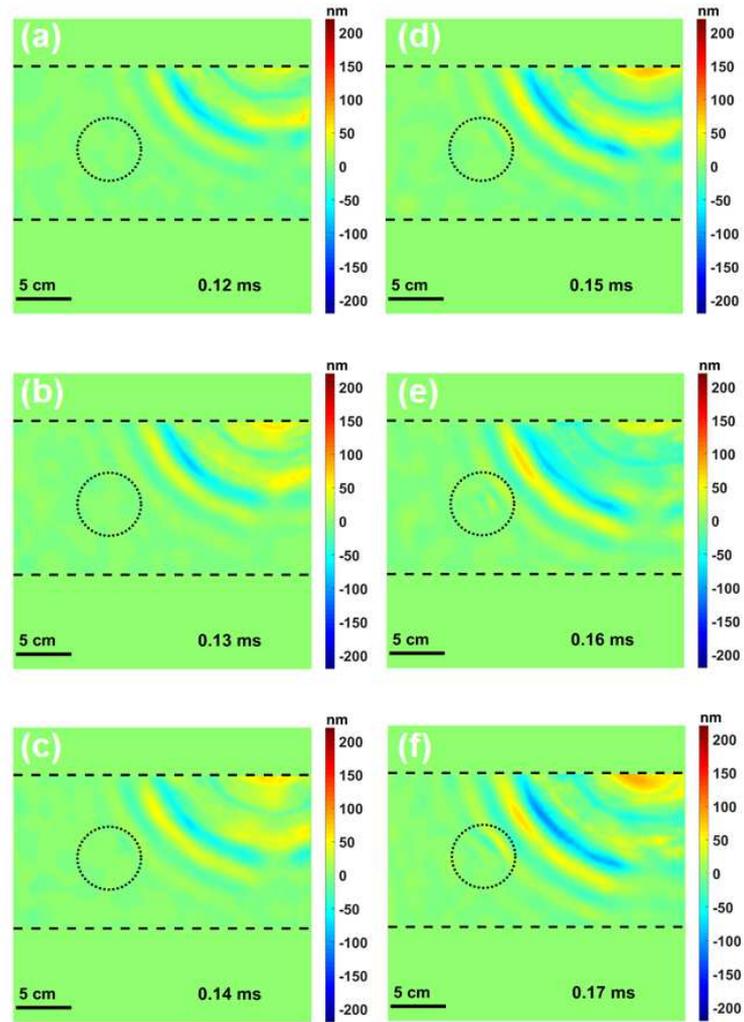
412 Fig. 9. Extracted modulo 2π phase maps over time interval [15.95ms;16.00ms] showing the vibration field with
 413 lower amplitude around the ABH, whereas the ABH center still exhibits phase jumps indicating local high
 414 vibration amplitude

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416 Supplementary material is provided through the movie of the wave propagating in the plate
 417 from which are extracted Fig. 5 to Fig. 9.

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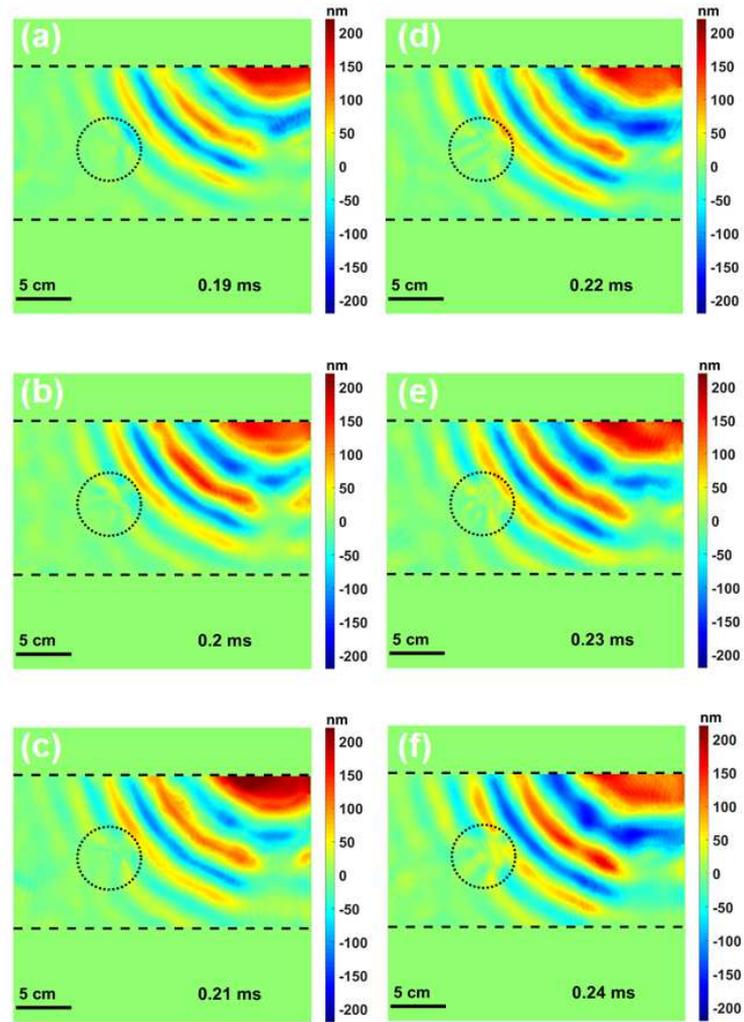
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Fig. 10. Amplitude maps corresponding to Fig. 6, over time interval [0.12ms; 0.17ms], showing the wave front
interacting with the screw inserted in the plate

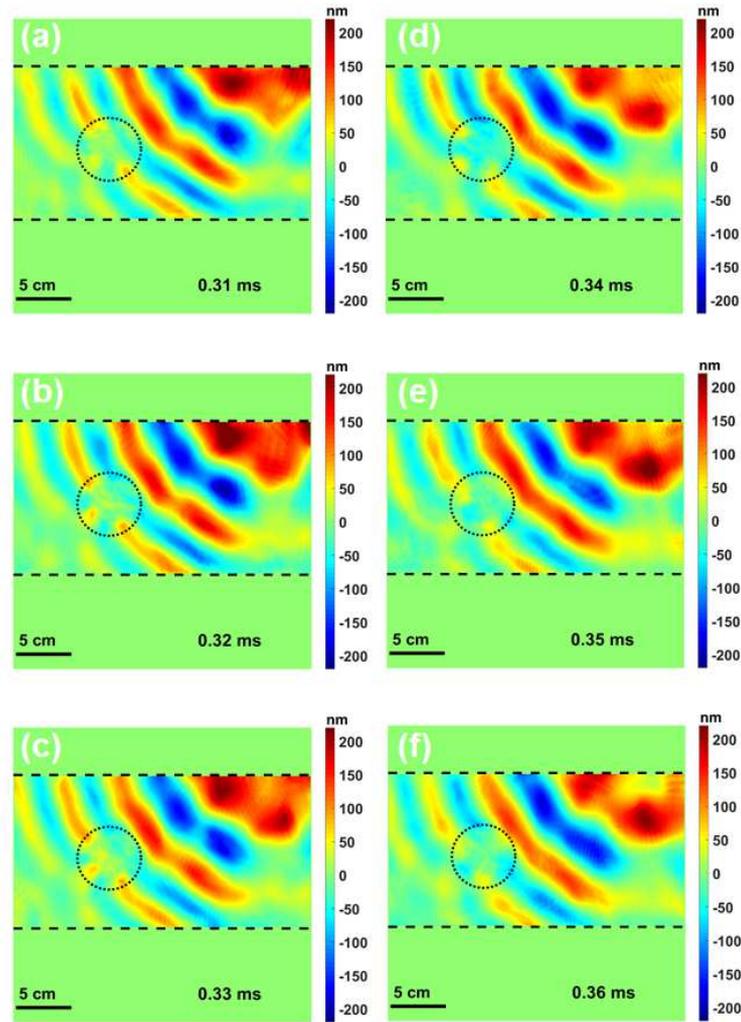
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Fig. 11 Amplitude maps corresponding to Fig. 7, over time interval [0.19ms; 0.24ms] showing the wave front interacting with the ABH area; the distortion of the wave front due to the velocity gradient is visualized



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Fig. 12. Amplitude maps corresponding to Fig. 8, over time interval [0.31ms; 0.36ms] showing the vibration field few instants after the wave front passed through the ABH area

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6. Conclusion

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This paper presents and **discusses the application** of a robust and efficient digital holographic set-up including a compact interferometer with the Fresnel configuration equipped with a

443 negative zoom and a diffractive optical element (DOE) to improve the photometric efficiency
444 of the set-up for illuminating large surfaces. We demonstrate the possibility of measuring
445 transient vibrations of structures at 100kHz frame rate when providing quantitative data on
446 380cm² rectangular spot at the object surface. This is the best performance ever achieved for
447 such area and frame rate, to the best of our knowledge. This paper provides the investigation
448 of traveling acoustic waves propagating in alloy plate equipped with a two-dimensional
449 Acoustic Black Hole (ABH). The wave front is generated by an impact hammer to yield 8N
450 force with duration about 50μs. The time sequence of the vibration field obtained after the
451 shock is depicted and exhibits the propagation of the wave front in the complex structure
452 made of the plate equipped with the ABH, and containing a screw. It follows that the
453 observation of the modification of the wave propagation can be observed at very short time
454 scale. Diffraction and reflection phenomena can be clearly seen when the wave front reaches
455 the screw. The modification of the wave front due to the gradient in elastic properties related
456 to the ABH area is also highlighted. Observations are well correlated to theoretical
457 approaches of literature.

458 The approach presented in the paper opens the way to a thorough analysis of physical
459 phenomena existing in and around the ABH, such as for example non-linear behavior in the
460 center of the ABH, or qualifying the efficiency of the wave trap by evaluating its reflection
461 coefficient. In addition, one might be able to address other problems which cannot be
462 addressed by classical experimental means such as vibrations of panels induced by hydro or
463 aero-acoustic sources, structural vibration induced by squeak and rattle noise.

464

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