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# Control analysis tools for active attenuation of vehicle longitudinal oscillations

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*This paper presents a set of control analysis to evaluate the quality of a given controller, in the context of driveline control. After having introduced the simplified mechanical model from which the analysis is performed, different indicators are presented. These indicators are chosen in order to give concise but essential information on the control system. In addition to the classical robustness margins, they evaluate some aspects of robust performance and the influence of the backlashes.*

**Automotive systems, Robust control, Vehicles and transportation systems,  $\mu$  analysis**

## I. INTRODUCTION

Embedded processors in actual vehicles allow to implement some "advanced" control strategy in order to improve vehicles behavior. However the lack of methodology restrains the use of such strategy and few paper demonstrates their interest in the context of vehicle dynamics control [9]. This study takes place in a more general project from PSA focusing on control methodology. The control design stage has to be more and more efficient to cope with an increasing complexity of the control strategy while the development period for a new vehicle design becomes shorter.

The vehicle driveline allows to transfer the engine torque to the wheels. Resonance in the elastic parts of the driveline are important to handle. In particular, one significant aspect of driveability is the attenuation of the first resonant mode which produces unpleasant (0-10 Hz) longitudinal oscillations of the car, known as shuffle. These oscillations are a cause of discomfort and generally occur during transient changes in driver demand (tip-in and tip-out) or on clutch engagement. The longitudinal oscillations are reduced by means of a controller acting on the engine torque. The problem of flexible transmission for electric systems has already been studied see Font and Brandenburg [7,2] for example.

A methodological effort was carried out in order to simplify the synthesis of this controller like pole placement or by  $H_\infty$  optimization. The low number and physical meaning of each tuning parameter are significant points to ensure the durability of a design method within the industrial field. It is now necessary to develop tools to evaluate the quality of a given controller. An effort will be brought on the simplicity and the relevance of the

indicators which will allow to discriminate two controllers. This is all the more significant since the objective notation of road handling and comfort of vehicles is difficult to obtain. These tools will allow to evaluate various aspects of robustness and control performance. The qualitative notation of the vehicle will thus be facilitated.

## II. MODEL OF THE POWERTRAIN

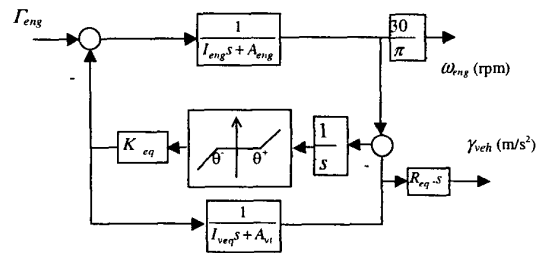


Fig. 1. Simplified model of powertrain

The model of the powertrain is a system of high order including many non-linearities. This model is too complicated for applying simple indicators. The model must be reduced. The modal reduction is not adapted because it doesn't keep a physical identity on each parameters. The mechanical reduction is more appropriate. The reduction of the complete model allow to obtain a model of order 3 and to preserve the main non-linearity. This model corresponds to an inertia in rotation involving another inertia by the way of a flexible connection and a threshold.

The input of the model  $\Gamma_{eng}$  (see figure 1) corresponds to the torque provided by the engine. The angular velocity  $\omega_{eng}$  of the 1<sup>st</sup> inertia  $I_{eng}$  is the image of the engine speed while  $\gamma_{veh}$  represents the vehicle longitudinal acceleration. Aerodynamics forces and the road profile are not considered.

Each parameter of this model is related to vehicle characteristics:

- $I_{eng}$  and  $I_{veq}$  (kg.m<sup>2</sup>) correspond respectively to the inertia of the engine flywheel and to the equivalent inertia of the vehicle.

- $K_{eq}$  (Nm/rad) is an aggregate stiffness related to the driveshafts, the clutch, the gearbox and the tires stiffness.
- $A_{enR}$  and  $A_{vt}$  (Nm.s/rad) include all existing frictions.
- $R_{eq}$  (m) is the equivalent radius of the wheel through the reduction of the gearbox and of the drive axle.
- $\theta^*$  and  $\theta$  (rad) are the angular bounds associated with the backlashes brought at the back gearbox input.

Some assumptions were made to establish this simplified model. It is supposed that the clutch is stuck and that the differential is blocked. The inertias of the gearbox driveshafts and of the wheels are neglected, thus the high frequency dynamics are not modeled.

Numerical values of each parameter are derived from the experimental data analysis and from information provided by the manufacturer. Each coefficient is uncertain. The frictions  $A_{enR}$  and  $A_{vt}$  are badly known. The value of the engine inertia  $I_{enR}$  is precisely known while the equivalent inertia of the vehicle  $I_{veq}$  can change depending on the vehicle load. The backlash is measured directly on the vehicle. It will be neglected during the control design which is based on the simplified model. Therefore, the engine speed  $\omega_{enR}$  and the vehicle acceleration  $\gamma_{veh}$  are related to the torque input  $T_{enR}$  by the two transfer functions:

$$\begin{cases} G(s) = \frac{\omega_{enR}}{T_{enR}} = \frac{30}{\pi} \frac{s^2 I_{veq} + s A_{vt} + K_{eq}}{D(s)} \\ G_{acc}(s) = \frac{\gamma_{veh}}{T_{enR}} = K_{eq} \frac{s R_{eq}}{D(s)} \end{cases} \quad (1)$$

$$\text{with } D(s) = s^3 I_{veq} I_{enR} + s^2 (I_{enR} A_{vt} + A_{enR} I_{veq}) + s (K_{eq} (I_{veq} + I_{enR}) + A_{enR} A_{vt}) + K_{eq} (A_{vt} + A_{enR})$$

These equations are representative of a system with a badly damped mode.

The purpose of the various design methods is to reduce the oscillations on vehicle acceleration. It consists in improving the damping ratio  $\xi_n$  of the function  $G_{acc}(s)$ . The strong constraint lies in the delay introduced by the four-stroke engine and its close control. It varies from 20 to 200 ms following the value of the engine speed. Moreover the robustness of the controller must be significant with regard to neglected dynamics and uncertainties on parameters. The numerical controller runs with variable step and is deduced according to Richard [14] from the continuous controller synthesized with the maximum delay. The delay margin must be higher than one period of sampling in the worst case.

Two design methods have successfully fulfilled this particular problem. The 1<sup>st</sup> one is based on an approach in term of pole placement [13] and the 2<sup>nd</sup> one is the H<sub>∞</sub> optimization [11]. Each proposed method presents a low number of independent tuning parameters.

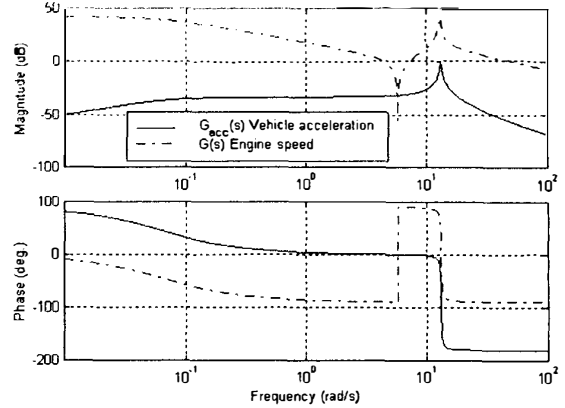


Fig. 2. Bode diagram of  $G(s)$  function engine speed and  $G_{acc}(s)$  function acceleration

### III. PRELIMINARY THEORETICAL RESULTS

#### A. The describing function method

The describing function method is well known for the analysis of the behavior of the closed loop system with a separable non-linearity. The analysis is based on the harmonic study of the signals in the loop (see figure 3). The assumption is made that the linear part of the system  $R(s)$  filters sufficiently the high frequencies so that only the 1<sup>st</sup> harmonic of the signal  $u = \Psi(y)$  is considered.

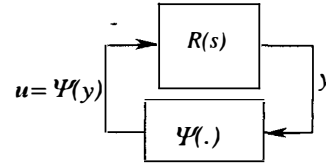


Fig. 3. Nonlinear feedback connection

This study is valid if the non-linearity  $\Psi$  is symmetrical compared to the origin and if it does not depend on time. Then, the non linearity can be replaced by his describing function defined by :

$$N(x_1, \omega) = \frac{M(x_1, \omega) + jQ(x_1, \omega)}{x_1} \quad \text{with}$$

$$\begin{cases} M(x_1, \omega) = \frac{2}{T} \int_0^T \Psi(x_1 \sin(\omega t)) \sin(\omega t) dt \\ Q(x_1, \omega) = \frac{2}{T} \int_0^T \Psi(x_1 \sin(\omega t)) \cos(\omega t) dt \end{cases}$$

$T$  and  $x_1$  are the period and the amplitude of the input signal  $y$ .

The non-linearity included in the model of the powertrain and allowing to take into account the looseness, is a threshold with an amplitude of  $\Delta = |\theta^* - \theta|$ . His describing function is real and corresponds to:

$$\begin{cases} N(x_1) = 1 - \frac{2}{\pi} \left( \arcsin\left(\frac{\Delta}{2x_1}\right) + \frac{\Delta}{2x_1} \sqrt{1 - \left(\frac{\Delta}{2x_1}\right)^2} \right) \\ \text{if } x_1 \geq \Delta/2 \text{ and } N(x_1) = 0 \text{ if } x_1 < \Delta/2 \end{cases} \quad (2)$$

The connection of the linear and nonlinear part will be unstable if the following relation is checked :

$$1 + N(x_1, \omega)R(j\omega) = 0.$$

Or if

$$R(j\omega) = -1/N(x_1, \omega) \quad (3)$$

$L(x_1, \omega) = -1/N(x_1, \omega)$  is called the critical locus. For the threshold, this locus corresponds in the Nyquist plan to the negative real axis lower than (-1). The Nyquist plot of  $R(j\omega)$  and the critical locus plot allow to know the points for which a risk of instability exists. Indeed the equation (3) is verified when  $R(j\omega)$  cuts  $L(x_1, \omega)$  in the Nyquist plan. The LOEB criterion is used to determine the stability of these points. The coordinates of the contact point provide information on the characteristics of the limit cycle.

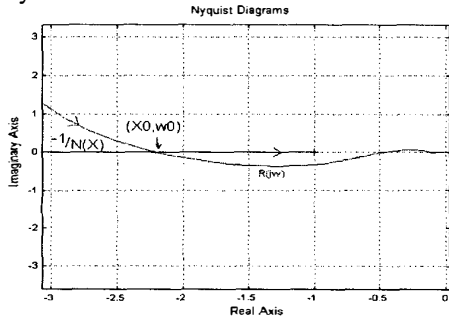


Fig. 4. Graphical method for detecting the existence of limit cycles

### B. $\mu$ analysis definition

The concept of structured singular value was first introduced by Doyle and Safonov [3,15]. Let us recall its definition and some properties:

$$\begin{cases} \mu_{\Delta}(M) = \left( \inf_{\Delta \in \underline{\Delta}} \left( \sigma(\Delta) / \det(I - M\Delta) = 0 \right) \right)^{-1} \\ \mu_{\Delta}(M) = 0, \text{ if } \forall \Delta \in \underline{\Delta}, \det(I - M\Delta) \neq 0 \end{cases}$$

where the set  $\underline{\Delta}$  is defined as follows:

$$\underline{\Delta} = \left\{ \Delta(s) = \text{diag} \left\{ \delta_1 I_{r_1}, \dots, \delta_r I_r, \varepsilon_1 I_{c_1}, \dots, \varepsilon_c I_c, \Delta_1(s), \dots, \Delta_q(s) \right\} \quad (4) \right. \\ \left. \begin{array}{l} \delta_i \in \mathfrak{R}; \quad \varepsilon_i \in \mathfrak{C}; \quad \Delta_i(s) \in \mathfrak{RH}_{\infty} \\ \vdots \end{array} \right.$$

$\mu_{\Delta}(M)^{-1}$  represents the smallest "size" of the matrix  $\Delta$  for which the matrix  $I - M\Delta$  becomes singular.

The  $\mu$  analysis requires a formalization of the uncertain system as in figure 5.

$F_u(H(s), 0) = F_1(P(s), K(s))$  represents the nominal closed loop system without the non linearity where  $P(s)$  is the synthesis model and  $K(s)$  is the controller.  $F_u(H(s), \Delta(s)) = T_{z,v}(s)$  is the disturbed closed loop system.

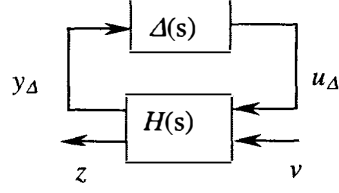


Fig. 5. Standard form for  $\mu$  analysis

### Small $\mu$ theorem (stability)

If  $H(s)$  has only left half plane poles, the system of figure 5 is stable for any uncertainty  $\Delta(s)$  of type (4) such that  $\|\Delta(s)\|_{\infty} < \alpha$  if and only if:

$$\forall \omega \in \mathfrak{R}^+ \quad \mu_{\Delta}(H_{y,\mu_{\Delta}}(j\omega)) \leq \alpha^{-1} \quad (5)$$

This theorem forms the basis of  $\mu$ -analysis. See [18] for the theory and [17,1] for instance, for the computation of  $\mu_{\Delta}$ .

This theorem can be formulated for the generalized stability see [6]:

If  $H(s)$  has only poles in the  $\Omega$  domain, the poles of the system of figure 5 will remain in the  $\Omega$  domain for any uncertainty  $\Delta(s)$  of type (4) such that  $\|\Delta(s)\|_{\infty} < \alpha$  if and only if:

$$\forall s \in \partial\Omega \quad \mu_{\Delta}(H_{y,\mu_{\Delta}}(s)) \leq \alpha^{-1} \quad (6)$$

where  $\partial\Omega$  is the border of the  $\Omega$  domain.

Here is an example of domain parameterized by a damping ratio  $\xi$ .

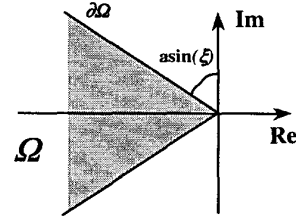


Fig. 6. Study area in the complex plan

If uncertainties are real,  $\mu_{\Delta}(H(s))$  can be discontinuous [12] and the maximum value of  $\mu_{\Delta}$  is difficult to find. The  $v$  analysis can be applied for avoiding the frequency grid see [4,8] for more details.

## IV. ANALYSIS TOOLS

The objectives of the feedback are:

- to preserve the stability of the system,
- not to modify the dominant real mode,
- to ensure a minimal attenuation of transmission oscillations despite the uncertainties on parameters.

Moreover, the parameters of the model vary or are badly known. It is thus significant to be able to evaluate the good behavior of the objectives with regard to all uncertainties with simple indicators.

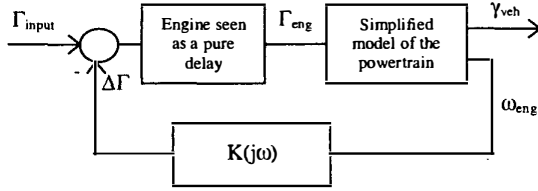


Fig. 7. Representation of the closed loop

The simplified model of the powertrain is represented in figure 1.

### A. Indicators of stability robustness

#### 1) Traditional margins

The gain, phase and module margins are significant indicators to guarantee the stability of the closed loop. The delay margin is representative of the robustness of the loop with regard to parasitic pure delays and neglected time constants. This point is all the more important to check that a variable delay is present in the loop.

#### 2) Dedicated margins

The parametric robustness is significant to check. We consider an uncertainty  $\Delta L$  on the system bounded by the pulsation  $\omega$  and the gain of the system  $L$  :

$$|\Delta L(j\omega)| < \beta\omega |L(j\omega)|$$

$L(j\omega) = K(j\omega)G(j\omega)$  is the transfer function in open loop.  $K(j\omega)$  is the controller and  $G(j\omega)$  is defined by the equation (1). According to the Nyquist criterion, stability will be ensured if uncertainty remains lower than the distance from  $L(j\omega)$  to the point (-1). We can then write that the system will be stable if

$$\forall \omega \in \mathbb{R}^+ \quad \beta\omega |L(j\omega)| < |1 + L(j\omega)|$$

Or if  $|T(j\omega)| < 1/\beta\omega$

$\beta$  can be calculated with the following expression:

$$\beta = \frac{1}{\sup_{\omega \in \mathbb{R}^+} \omega |T(j\omega)|} = \|s.T(s)\|_{\infty}^{-1} \quad (7)$$

It should be noted that  $\beta$  is the dynamic margin  $T_{dyn}$ , introduced by de Larminat [10] which gives an approximation of the delay margin. A great value of  $T_{dyn}$  will be the proof of a good robustness.

### B. Nominal performance indicators

The reduction of the acceleration oscillations is equivalent to reduce the maximum value of the frequency response of the transfer function acceleration  $G_{acc}(s)$  defined in equation (1) (see figure 2). For the nominal system, the reduction ratio  $\alpha_r$  of this peak is representative of the level of performance. Its expression is given by:

$$\alpha_r = \frac{\log\left(\frac{\|G_{acc}S\|_{\infty}}{\|G_{acc}\|_{\infty}}\right)}{\log(2\xi_a)} \quad (8)$$

where  $\xi_a$  is the damping ratio of the pole responsible for the resonant peak of the acceleration function  $G_{acc}(s)$  in open loop.

The dynamics of the system must not be modified. The controller must intervene the least possible in the frequential bandwidth. The control is supposed active when the input torque  $\Gamma_{input}$  is modified of more than 10% by the corrective torque  $\Delta\Gamma$  (see figure 7). This can be expressed by the following relation:

$$\left|\frac{\Delta\Gamma}{\Gamma_{input}}\right| = |T| \geq 0,1$$

We introduce  $B_{10} = \omega_{M10} - \omega_{m10}$  where  $\omega_{M10}$  and  $\omega_{m10}$  are defined by:

$$\begin{cases} \omega_{M10} = \sup_{\omega \in \Omega_{10}} \{\omega\} \\ \omega_{m10} = \inf_{\omega \in \Omega_{10}} \{\omega\} \end{cases} \quad \Omega_{10} = \{\omega / |T(j\omega)| \geq 0,1\} \quad (9)$$

The greater  $B_{10}$ , the wider in frequency domain the action of the controller is.

The engine speed is used for the feedback (see figure 7). This data is measured on the engine flywheel, source of parasitic noises. The closed loop sensitivity to the measurement noises can be analyzed with the help of the indicator  $N$  introduced by de Larminat [10]:

$$N = \frac{\|S(s)S_u(s)\|_2}{\|S_u(s)\|_2} \quad (10)$$

$S_u(s)$  is the command sensibility function defined by  $K(s)S(s)$  and  $S(s)$  is the sensibility function of the loop. The function  $S(s)S_u(s)$  characterizes the capacity of the closed loop system to transmit the noises when the control is active, while  $S_u(s)$  is representative of the total transmission of the noises. The lower  $N$ , the less sensitive the closed loop is to the measurement noises.

### C. Robust performance indicators

#### 1) Calculation of guaranteed damping ratio

The small  $\mu$  theorem on the generalized stability, presented in part III, is applied to calculate the damping ratio which is guaranteed for the worst case of parametric uncertainties. The area study  $\Omega$  is represented in figure 6.  $\Omega$  is parameterized by the damping ratio  $\xi_g$  and the border  $\partial\Omega$  is given by :

$$\partial\Omega(\xi_g) = \left\{ -\xi_g \omega_n \pm j\sqrt{1 - \xi_g^2} \omega_n / \omega_n \in \mathbb{R}^+ \right\}$$

$\xi_g$  is calculated by dichotomy. The  $\mu$ -analysis on a border  $\partial\Omega(\xi_i)$  provides a value of  $\mu_{max}$ . If  $\mu_{max}$  is lower than the desired value,  $\xi_i$  is increased and  $\mu_{max}$  is calculated with the new value of  $\partial\Omega(\xi_i)$ . The calculus is executed until the desired precision on  $\xi_i$  is obtained. This method allows to guarantee a minimum damping ratio in relation to the wished uncertainties intervals.

Inversely, the small  $\mu$  theorem can be applied to determine the variation intervals of parameters which will ensure the stability.

## 2) Nonlinear analysis

The describing function method gives good results on this model. But the result is only valid for the set of parameters considered. The concept of parametric robustness misses.

To fill this lack, the robustness margin introduced by Ferreres [5] is used. It consists in calculating the size of the uncertainties which lead to a given value of the limit cycle. The study diagram is the following :

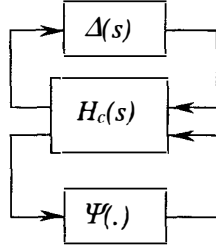


Fig. 8. Study diagram for the calculation of the robustness margin

The non-linearity  $\Psi$  is represented by his describing function  $N(X, \omega)$  defined in ( 2 ). There will be existence of a limit cycle if the following equation is verified :

$$\det(I - F_u(H_c(j\omega), \Delta)N(X, \omega)) = 0.$$

The margin is then defined as follows :

$$r(X, \omega) = \inf_{\Delta \in \Delta} (\|\Delta\|) / \det(I - F_u(H_c(j\omega), \Delta)N(X, \omega)) = 0.$$

It is homogeneous to the inverse of a singular value.

The weaker the margin  $r(X, \omega)$ , the closer to the limit cycle of characteristic  $(X, \omega)$  is the system.  $r(X, \omega)$  characterizes the distance from the closed loop system to the cycle limit  $(X, \omega)$ . It allows to deduce the way in which the limit cycle will be modified when the parameters of the model change.

## V. APPLICATION

### A. Controllers design

Two methods will be compared : the robust pole placement method [14] and  $H_\infty$  optimization [11]. The two methods are based on an iterative design. All the objectives are not taken into account a priori, this being explained partly by the reduced number of tuning parameters. The same model of synthesis is used for calculating the 3 regulators with a delay of 0,2 s approximated by a 4 order Pade function. The aim of the feedback is to reduce the oscillations and to assure the biggest parametric robustness. All the controllers obtained are stable.

#### 1) $H_\infty$ method

The method is based on frequential weightings on sensibility functions. Two independent tuning parameters are sufficient to obtain the objectives, see [11] for a detailed methodology. The 1<sup>st</sup> controller called type I is synthesized with a direct method. The reduction of the oscillations is ensured by the 1<sup>st</sup> tuning parameter via the

transfer function  $G_{acc}(s)S(s)$ . The 2<sup>nd</sup> parameter guarantees the complementary module margin via the transfer function  $T(s)=1-S(s)$ . These constraints don't avoid the cancellation poles-zeros between the system and the controller.

The type II controller is obtained with the same tuning parameters but the criterion is widened. It takes into account a weighting on the transfer function  $K(s)S(s)$ . The parametric robustness is increased and it avoids the cancellation poles-zeros.

#### 2) Robust pole placement method

The method constraints the closed loop poles to be in a particular locus in the complex plan. The dominant real pole is kept. A first part of the closed loop poles are deduced from the open loop poles projection on the vertical  $-1/T_f$ . The second part of the poles are calculated with the open loop zeros previously stabilized and damped by the desired damping ratio  $\xi_d$  (see [13]).

### B. Results

The guaranteed damping ratio is calculated on realistic intervals of parameters.

Here is the comparative analysis of all the indicators previously presented, applied to the model and the 3 different regulators :

	$H_\infty$ controller type I	$H_\infty$ controller type II	RPP controller
Order of the controller	9	9	7
Gain margin (dB)	8,38	8,4	11,6
Phase margin (°)	63,53	65,6	73,4
Module margin	0,602	0,61	0,71
Delay margin (ms)	+68,7 / -123,1	+76 / -122,4	+86,3 / -114,7
$T_{dvn}$	$62,6 \cdot 10^{-3}$	$71,2 \cdot 10^{-3}$	$79,4 \cdot 10^{-3}$
Peak reduction of $G_{acc}S$	81,2%	69,6%	68,1%
$B_{10}$ (rad/s)	67,7	91,7	35,7
$N$	1,27	1,01	1,13
Guaranteed $\xi$	None	0,175	0,16
	$\omega_{NL} = 5,75$	$\omega_{NL} = 6,44$	$\omega_{NL} = 6,92$
Characteristics of the limit cycle (rad/s)	$x_{NL}/\Delta =$ 395,48	$x_{NL}/\Delta = 2,25$	$x_{NL}/\Delta = 2,68$

The indicators allow to turn down the regulator of type I. Indeed it has the lowest robustness with regard to parametric uncertainties. Its noise sensitivity is comparatively worse. It does not guarantee any performance and the characteristics of the limit cycle are unreasonable. This controller didn't intentionally fill the specifications. It is confirmed and proves the accuracy of the indicators.

On the other hand the indicators do not allow to decide between type II and RPP controller. Each one has

advantages in his field. The RPP controller presents better margins, but type II controller is more robust compared to the parametric variations and has a weaker limit cycle. The application of the robustness margin gives an additional indication to evaluate the quality of the two controllers.

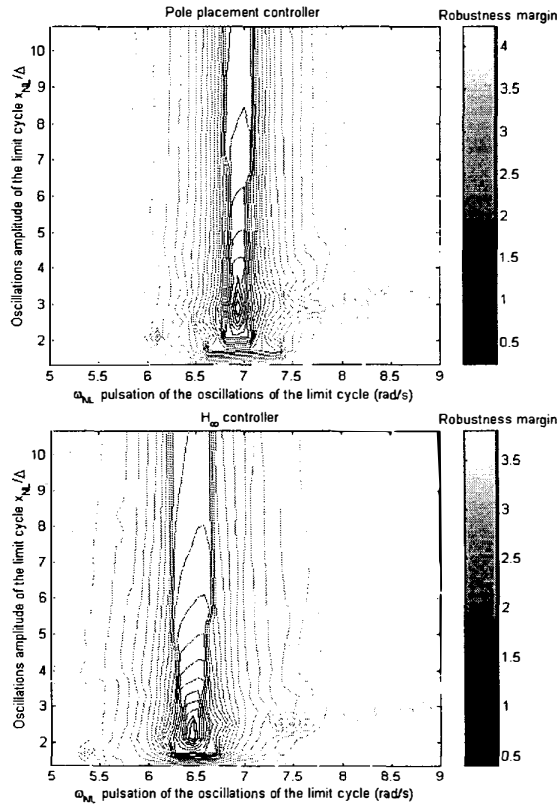


Fig. 9. Robustness margin for pole placement controller and  $H_\infty$  type II controller

The dark lines on the figure 9 correspond to low value of robustness margin. It confirms the results of the describing function method. Indeed, the margin is very low near the characteristics of the limit cycle found previously.

If the parameters are modified, the size of the uncertainty increases and the characteristics of the limit cycle change: the pulsation  $\omega_{NL}$  will vary in little proportion whereas the amplitude  $x_{NL}$  depends largely on the parametric variation.

The pole placement controller tolerates weaker variation on parametric uncertainties in comparison with the  $H_\infty$  controller.

## VI. CONCLUSION

Some control quality indicators have been proposed in the context of the vehicle driveline oscillations attenuation. They concentrate information obtained from a detailed frequency analysis but don't require to be an expert to be understood. Their examination will constitute for control designer in PSA a determinate stage before making more extensive simulation or experimental tests.

Some of the proposed indicators are classical while some others take advantages of recent development in  $\mu$  or  $v$ -analysis. Some are "universal" while others are more or less dedicated to the controlled system considered here. Nevertheless, each presented indicator can be extended to the multivariable control case (see Scorletti [16] for example for multivariable delay margins). This is an important feature because future vehicle will include multi-torque providers in addition to the four-stroke engine (electric motor, piloted clutch) leading to multivariable control problem.

Finally, the proposed control analysis tools complete the methodological effort made in [13,11] for model reduction and control design.

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