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PHYSICAL SIMPLIFICATION OF MIMO LINEAR MODELS FROM ELEMENTARY COMPONENTS MANIPULATIONS

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Abstract: Numerous research efforts have focused on the development of tools for model reduction. Most of these efforts resulted in mathematical methods for model reduction which suffer from inadequate physical interpretations. The problem of simplified physical model design is just emerging as a major research area. Rare but interesting preliminary contributions address the model reduction issue from a physical point of view. However, they are more or less disconnected from the classical literature and tools dealing with model reduction. This paper considers the problem of simplifying physical models under the constraint of keeping the physical meaning. This is done in making use of the general principles of Bond Graphs together with the gramians as analysis tools. The method proposed is applied to the design of a low frequency vehicle driveline model.

Keywords: Model Reduction, Physical Models, System Analysis, Methodology, Bond Graphs

1 INTRODUCTION

A problem closely related to the plant modeling issue is controller complexity. Complex plant models will usually lead to high order controllers and difficulties to implement them. On the assumption that robust control deals with “modeling gap”, it would seem reasonable to consider the possibility to replace complex plant models with simpler ones. Indeed the model reduction is an integral part of robust control system design. In one hand simple models simplify the understanding of design limitations. In the other hand the controller synthesis is faster, requiring less storage, is easier to implement and is more robust because a lower degree.

Until relatively recently, reduction model was often based on physical intuition, for example removing high frequency modes of flexible structures or neglecting electrical dynamics. Important advances have been made over the past 20 years providing efficient tools based on state space truncation, singular perturbation and special realizations such as the balanced form. In that case, the gramians, through the Hankel singular values, provide a powerful analysis tool, giving *a priori* bound on the reduction model error H_{∞} -norm (Obinata and Anderson, 2001). However, due to the prior change of coordinates, these methods do not preserve the physical meaning of the model. This is their major drawback.

Indeed keep the physical meaning is particularly interesting (Chevrel, 2003). The legibility of the controller may be

improved (for example based on a state estimate feedback law, see Karnopp (1979)) which can facilitate its tuning and maintenance. Moreover, a physical model is interesting both to perform a robustness analysis and to design the plant itself (constituent part, sizing) through a parametric sensitivity analysis.

Nevertheless preserve the physical meaning is a constraint for the reduction process, preventing usual changes of coordinate. Moreover a physical representation is required to explicit the system structure. In this way, one well-known approach to deal with multi-domain engineering problems is the Bond Graph representation elucidated by H. Paynter (1961).

Some preliminary works have already considered the model reduction issue from a physical point of view. The name of “proper model” has been introduced in 1995 by B.H. Wilson, to designate the “just enough accurate” physical model (Roboam, 2001). The method called MORA (Model Order Reduction Algorithm), an energy-based procedure (Loucas, *et al.*, 1998b, 1999), seems to be one of the most promising way to deal with physical model analysis and reduction. (See also Dauphin-Tanguy, *et al.*, (1985), Sueur and Dauphin-Tanguy (1991) and Gandanegara, *et al.* (2001) for details about the singular perturbation method applied to Bond Graph and Gandanegara, *et al.* (2003) for a comparison of this method with MORA). In MORA, the reduced model is obtained suppressing less energetically active elements. This paper pursues the same objective but using linear fractional

representations (Zhou, *et al.*, 1996; Magni, 2001), associated to a physical interpretation, and a gramian based analysis. In this sense, it bridges the gap between classical algebraic model reduction and current physical model reduction techniques.

The presentation of this paper is as follows. Based on the basic principles of bond graph, the “component graph” is first introduced in section II. It is then shown in section III how to obtain state space and linear fractional representations from it. Section IV considers the problem of components manipulation to obtain reduced order models. As an example, the resulting procedure is applied in section V in order to design a low frequency simplified model of a vehicle driveline.

II. FROM BONDGRAPH TO COMPONENT-GRAPH

II.1 Physical modeling and Bond Graphs

A physical system can be modeled in a unified way and analyzed by using the bond graph formalism (Paynter, 1961; Karnopp, *et al.*, 2000). This formalism uses the effort-flow analogy to describe physical processes (Hezemans and Geffen, 1991). Effort and flow are termed *power variables* (e , f) and their product is *power*. In addition to the power variables, there are also generalized variables (p , q) obtained by integration over time and called *impulse* and *displacements*. All physical models can be described using several basic elements : sources (S), detectors (D), storages (I and C), dissipations (R), connections (transformers TF and gyrators GY). These elements are connected to junction elements (0 or 1) with bonds linked to power variables (e and f) going through.

Furthermore, the Bond Graph theory enables to establish the causality structure of the model. The sequential causal assignment procedure, presented in Karnopp, *et al.*, (2000), guarantees to obtain a right one. The bond graph to which causality has been assigned in that way is termed a causal bond graph. The Bond graphs with complete causality assignment can be directly put in a state-space form, if storage elements have integrate causality and if there is no algebraic loops (Karnopp, *et al.*, 2000; Rosenberg, 1971).

Let us now introduce the “component graph”. Less general than the bond graph it is advantageous in that it gives a unique energetic interpretation of the physical process. To obtain such a unique representation, the following assumptions are made:

A1: the causality is fixed following the sequential assignment procedure with the restriction to have only integral causality in the storage elements;

A2: gyrators are not allowed.

The consequences of these assumptions are:

1. The resulting constrained bond graph is unique and has a clear physical meaning.

2. The connection of components of the same type is not possible, except aggregating them (which is particularly important for model simplification).

3. A unique convention between different physical domains

has to be applied.

4. Cinematic constraints can't be represented with usual gyrators series. Nevertheless a possible solution could be the gyrator use (Favre, 1997).

II.2 The component graph

Under assumptions **A1** and **A2**, the basic elements S , D , I , C and R are linked in a unique manner through a multiport junction of type 0 or 1 , in order to generate the multiport components presented in table 1. To generate a model with such components and preserving **A1** and **A2**, it is necessary to add the constraint **C1**. See Ripert (2003) for an associated graphical formalism description.

C1: components including a 0 (rec. 1) junction have to be connected to components including a 1 (rec. 0) junction only.

Table 1: Basic components

	Effort	Flow
Source	Se ($S + 0$)	Sf ($S + 0$)
Detector	De ($D + 1$)	Df ($D + 1$)
Storage	I ($I + 1$)	C ($I + 1$)
Dissipation	R' ($R + 1$)	R ($R + 0$)

III. LINK BETWEEN LINEAR FRACTIONAL REPRESENTATION AND COMPONENT-GRAPHS

III.1 Algebraic relations

In one hand, the constitutive laws of all the components may be synthesized by the algebraic formulation:

$$\begin{pmatrix} w_i \\ w_c \end{pmatrix} = \begin{pmatrix} \Delta_i s^{-1} & \\ & \Delta_c s^{-1} \end{pmatrix} \begin{pmatrix} z_i \\ z_c \end{pmatrix}, \quad \begin{pmatrix} w_R \\ w_{R'} \end{pmatrix} = \begin{pmatrix} \Delta_R & \\ & \Delta_R \end{pmatrix} \begin{pmatrix} z_R \\ z_{R'} \end{pmatrix},$$

$$\begin{pmatrix} w_{S'} \\ w_{S''} \end{pmatrix} = \begin{pmatrix} u_i \\ u_c \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} y_o \\ y_i \end{pmatrix} = \begin{pmatrix} z_{D'} \\ z_{D''} \end{pmatrix} \quad (1), (2), (3), (4)$$

With:

w_X : input power variable of component of type X

z_X : output power variables of component of type X

Δ_X : diagonal matrix constituted by the operator describing the constitutive law of component of type X

s^{-1} : integral operator

I_d : identity matrix

In the other hand, the connections between the power variables w and z of each component may be formulated as: (5)

$$\begin{pmatrix} z_i & z_c & z_R & z_{R'} & z_{D'} & z_{D''} & z_{S'} & z_{S''} \end{pmatrix} = \mathcal{M} \begin{pmatrix} w_i & w_c & w_R & w_{R'} & w_{S'} & w_{S''} & w_{D'} & w_{D''} \end{pmatrix}$$

where \mathcal{M} is named the connection matrix.

Remark: Each element \mathcal{M}_{ij} corresponds to the weight and the orientation of the flux connection between the j^{th} component and the i^{th} . Each connection is bi-directional with an effort at the opposite direction of the flux. So \mathcal{M} is antisymmetric.

III.2. LFR representation

Because exogenous components (components of type S_f , S_e , D_e and D_f) have only one port variable (input or output, depending on sources or detectors), the matrix \mathcal{M} may be simplified suppressing appropriate columns (resp. lines). Then for an index sorted to match the decomposition $(I \ C \ R^T \ R^T \ S_f \ S_e \ D_e \ D_f)$, the matrix \mathcal{M} has the form: (6)

$$\mathcal{M} = \begin{pmatrix} 0 & \mathcal{M}_{1c} & 0 & \mathcal{M}_{1r} & 0 & \mathcal{M}_{1e} \\ -\mathcal{M}_{1c} & 0 & \mathcal{M}_{1p} & 0 & \mathcal{M}_{1y} & 0 \\ 0 & -\mathcal{M}_{1p} & 0 & \mathcal{M}_{1r} & 0 & \mathcal{M}_{1e} \\ -\mathcal{M}_{1c} & 0 & -\mathcal{M}_{1p} & 0 & \mathcal{M}_{1y} & 0 \\ 0 & -\mathcal{M}_{1p} & 0 & -\mathcal{M}_{1r} & 0 & -\mathcal{M}_{1e} \\ -\mathcal{M}_{1c} & 0 & -\mathcal{M}_{1p} & 0 & -\mathcal{M}_{1y} & 0 \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} \\ \mathcal{M}_{31} & \mathcal{M}_{32} & \mathcal{M}_{33} \end{pmatrix}$$

in order to match: (7)

$$\begin{pmatrix} \tilde{z}_1 & \tilde{z}_2 & \tilde{z}_3 & \tilde{z}_4 & \tilde{z}_5 & \tilde{z}_6 \end{pmatrix}^T = \mathcal{M} \begin{pmatrix} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 \end{pmatrix}^T$$

where: $\mathcal{M}_{Y \rightarrow X}$ denotes the flux connections between the components class Y to X ; the null matrices rely to the interdiction to connect two components with incompatible ports.

By applying (1) (2) (3) and (4) to (7), an input-output relationship is obtained:

$$\begin{pmatrix} y_v \\ y_f \end{pmatrix} = F_u(\mathcal{M}, \Delta(s)) \begin{pmatrix} u_f \\ u_v \end{pmatrix} \quad (8)$$

Let us define the transfer matrix $P(s)$ and the matrix Δ :

$$P(s) := \begin{pmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix} \triangleq \begin{pmatrix} \mathcal{M}_1 & \mathcal{M}_2 & \mathcal{M}_3 \\ \mathcal{M}_4 & \mathcal{M}_5 & \mathcal{M}_6 \\ \mathcal{M}_7 & \mathcal{M}_8 & \mathcal{M}_9 \end{pmatrix} \quad (9)$$

$$\Delta = \text{diag}(\Delta_1, \Delta_2, \Delta_3, \Delta_4) \quad (10)$$

Theorem 1 :

A bond graph representation under assumptions $A1$ and $A2$, built according to the constraint $C1$ can be associated in a one to one manner to the following Linear Fractional Representation (LFR):

$$\begin{pmatrix} y_v \\ y_f \end{pmatrix} = F_u(P(s), \Delta) \begin{pmatrix} u_f \\ u_v \end{pmatrix} \quad (11)$$

Under the assumption that the bond graph representation is minimal in the I , C , R elements, this LFR is minimal in the physical parameters appearing in Δ .

Proof: Under the assumptions $A1$ and $A2$, a Bond Graph can be represented as a component graph (§ II-2) which can be written as the LFR of Fig. 1. (equivalent to (11)). The parameters of type I , C , R or R^T appear as many times as there are arc components or equivalently elements in the initial bond graph (§ III-2). The Bond Graph being minimal (in the number of elements), the LFR (11) is minimal in parameters.

Remark 1: A *a posteriori* LFR reduction is a difficult task (Beck and D'andrea, 1998; Chevrel, 1993). The previous method

avoids this one leading to a *minimal* LFR in a systematic way.

Remark 2: The LFR (11) is derived directly from the interconnection matrix \mathcal{M} which special form guarantees the *physical* interpretation of the model (i.e satisfying $C1$ and consequently $A1$ and $A2$). This kind of LFR will be denoted *Physical* LFR (PLFR). The aim of the model reduction method proposed in this paper will be to get a reduced PLFR.

Remark 3: The concept of PLFR may be extended without restriction to components with non linear constitutive laws

III.3. Parameterized state-space forms

A state space realization may be easily derived from the PLFR. The system matrix choosing the generalized variables (§ II.1) as state variables:

$$\begin{pmatrix} A(\Delta) & B(\Delta) \\ C(\Delta) & D(\Delta) \end{pmatrix} = \begin{pmatrix} A\Delta_R & B_2 \\ C_1\Delta_K & D_{22} \end{pmatrix} + \begin{pmatrix} B_1 \\ D_{21} \end{pmatrix} \Delta_{R'R} (I - D_{11}\Delta_{R'R})^{-1} (C_1\Delta_K \ D_2) \quad (12)$$

IV. COMPONENT-REPRESENTATION ANALYSIS AND MANIPULATIONS

A3: the system considered from now is stable.

The objectives of this section are twofold : 1/ define tools allowing to analyze the relative contribution of each component into the input/output behavior considered (§ IV.1); 2/ propose a methodology to suppress the components of lower contribution to get a simplified physical model (§ IV.2).

For system analysis it is useful to define the output matrices of all the components:

$$\begin{aligned} C_1(\Delta) &= \begin{pmatrix} C_{1c}(\Delta) \\ C_{1r}(\Delta) \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & (I - D_{11}\Delta_{R'R})^{-1} C_1\Delta_K \end{pmatrix} \\ D_2(\Delta) &= \begin{pmatrix} D_{2c}(\Delta) \\ D_{2r}(\Delta) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & (I - D_{11}\Delta_{R'R})^{-1} D_{12} \end{pmatrix} \end{aligned} \quad (13)$$

The *dual system*, consisting to "invert" the part of the input and the output, may also be useful for analysis purposes (remember that controllability and observability are dual properties). It can be defined from the dual PLFR transposing the connection matrix in (6):

$$\begin{pmatrix} u_f \\ u_v \end{pmatrix} = F_u(\mathcal{M}^T, \Delta(s)) \begin{pmatrix} y_v \\ y_f \end{pmatrix} \quad (14)$$

In that case, the output matrices of the component form are: $\bar{C}_1(\Delta)$ and $\bar{D}_2(\Delta)$, obtained in the same way than $C_2(\Delta)$ and $D_1(\Delta)$ by applying equation (13) to (11).

Usually, the reduction methods make use of gramians to analyze the contribution of each state variable. Indeed, gramians are very interesting tools to sum up the controllability and observability of each state variable. The idea here is to generalize this analysis to others latent variables: the power variables of each component.

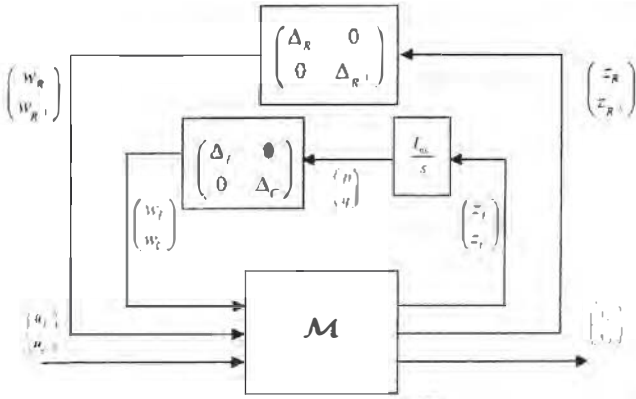


Fig. 1. Expanded form of the Physical LFR

IV.1. Component energy and interaction analysis through gramians

Consider the stable system $\Sigma(s) := (A \ B \ C \ D)$ with (u, x, z) its input, state and output variables respectively.

1. The (controllability) gramian W of $\Sigma(s)$ is solution of the Lyapunov equation : $AW + W.A^T + B.B^T = 0$.
2. W is a covariance matrix into the Hilbert space L_2^2 . Indeed let $x(t)$ be the normalized impulse state-response of $\Sigma(s)$,

$$\text{then: } \int_0^{\infty} x(t)x^T(t) dt = W.$$

3. The covariance matrix E of the system output is obtained from W as : $E = C_+ W C_+^T$.
4. Let W_{11} and W_{22} be the gramians associated to the systems $\Sigma_1(s)$ and $\Sigma_2(s)$. The gramian W of the product $\Sigma(s) = \Sigma_1(s) \cdot \Sigma_2(s)$ can be written as : $W = \begin{pmatrix} W_{11} & W_{12} \\ W_{12}^T & W_{22} \end{pmatrix}$.

Let us now define:

- $F(s)$ a filter used to define frequency specifications,
- $\Sigma_C(s) := (A(\Delta) \ B(\Delta) \ C(\Delta) \ D(\Delta))$ the transfer between system inputs and components outputs obtained from (12) & (13),
- $\Sigma_O(s) := (\bar{A}(\Delta) \ \bar{B}(\Delta) \ \bar{C}(\Delta) \ \bar{D}(\Delta))$ the transfer between system outputs and components outputs obtained from (14).
- W_C and W_O the gramians of $F(s) \cdot \Sigma_C(s)$ and $F(s) \cdot \Sigma_O(s)$ respectively.
- E_C and E_O , the covariance's matrices of the outputs of Σ_C and Σ_O respectively.

E_C and E_O give insight on how the components contribute to the input output relation. On one hand, the diagonal elements of E correspond to the square of the H2 norm of each

component output. Roughly speaking they quantify the *proper activity* (output signal energy) of each component. On the other hand, the non-diagonal elements, obtained by integration of a product of a flow and effort variables, can be interpreted as *energy*. Thus the elements E_{ij} can be used to quantify the *energy link* between the components indexed i and j . Based on this interpretation, two normalized balanced indicators (taking into account both output and input structures) will now be associated to each i^{th} component : α_i and β_i .

Assuming that E_{CO} denotes the product $E_{CO} = E_C \cdot E_O$, α_i and β_i are defined as:

$$0 \leq \alpha_i = \frac{2 \cdot (E_{CO})_{ii}}{\sum_{k=1}^n (|(E_{CO})_{ki}| + |(E_{CO})_{ki}|)} \leq 1 \quad (15)$$

$$0 \leq \beta_i = \frac{|(E_{CO})_{ii}|}{\sum_{n=1}^n |(E_{CO})_{ij}|} \leq 1 \quad (16)$$

α_i quantifies the energetic dependency of the component number i to all others.

β_i quantifies the relative activity of the component indexed i , relatively to the activity of all other components.

The following criteria are defined to evaluate the relative importance of the i^{th} component in the system behavior.

Conjecture (suppressibility conditions):

1. A component can be suppressed if it is energetically independent from the others and its proper energy is low.
2. A component can be suppressed if it is energetically totally dependent from the others.

Proposition (suppressibility criterions) :

The constitutive law of the i^{th} component will be said suppressible if one of the following criterion is satisfied:

$$(\alpha_i \approx 1 \text{ and } \beta_i \approx 0) \text{ or } (\alpha_i \approx 0 \text{ and } \beta_i \approx 0). \quad (17)$$

Remark: The first criterion generalizes the reduction concept used in the balanced truncation method. (Obinata and Anderson, 2001). In the theoretical case of a balanced realization, the property : $\forall i, \alpha_i = 1$ is verified. Then the i^{th} Hankel singular value can be neglected toward the others and truncated if it verifies: $\beta_i \approx 0$.

IV.2 Physical model simplification

To guaranty the uniqueness of the result, simplification rules are specified. The different possible situations are classified depending on the types of the components connected to the one to be suppressed (*neighborhood*). Firstly, all the junctions of the *neighborhood* are concatenated into one named the *new junction*. Then, different cases can be distinguished:

1. *If there is only one type of components in the neighborhood.*

The sum of the constitutive laws of the *neighborhood* are added up and associated to the *new junction*.

2. If there are R or R^{-1} components and only one other type of components in the neighborhood. The constitutive laws of this last type of component are added up and associated to the new junction. The constitutive laws of R or R^{-1} components are summed, next inversed and associated to a junction connected to the new junction.
3. Other situation: In this case, system inputs or outputs are necessarily present in the neighborhood. They are neglected until to be in one of the previous cases.

Contrary to Loucas, *et al.*, (1997), no extensive simulations are required to apply the proposed procedure. Moreover its theoretical foundation seems to the authors to be clearer than previously (Loucas and Stein (1999)).

V. VEHICLE DRIVELINE MODEL SIMPLIFICATION

In a vehicle, the driveline allows the transfer of engine torque to the wheels. Resonance in the elastic parts of this one have to be handled carefully. In particular, one significant aspect of driveability is the attenuation of the first torsional mode which produces unpleasant (0 to 15 Hz) longitudinal oscillations of the car, known as shuffle. The model considered here is linear of order 13. It includes parallel connections because of the engine mounting and has two inputs : the engine (F_{eng}) and the starter (F_{sta}) torques (in this application, the starter is an electrical motor located into the clutch). The associated technological representation is given in figure 2.

The specifications for the simplification of the model are as follows. The acceleration of the vehicle is the output of interest. The simplified model must be accurate in the frequency range 0.2 to 16 Hz (corresponding to driveability requirements).

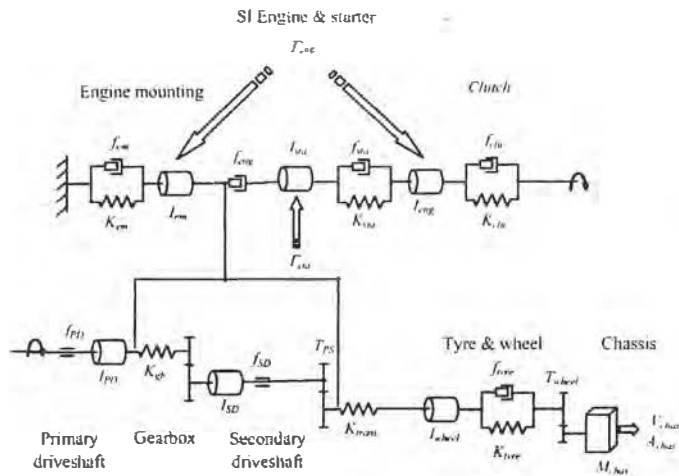


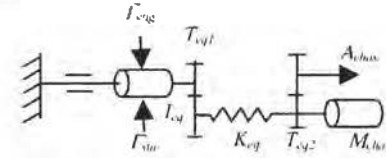
Fig. 2. Technological representation of the driveline model

The frequency specifications are taken into account thanks to the passband filter $F(s)$ (§ 4.1). Making use of (11) and applying iteratively the model simplification rules presented in §IV.2, according to criteria given table 2, lead to the simplified

model presented in figure 3. Because the simplified model has the two inputs applied to the same inertia, the two transfers of the simplified model are identical. Considering the chassis acceleration as output, the frequency magnitude responses of the reduced and initial models are superimposed in figure 4. They are very similar in the frequency range considered, showing the quality of the simplified model. Moreover, the analytic dependency between the components of the resulting model and those of the initial ones is made explicit (fig. 3).

Table 2: Iterately removed components

Suppressed component	α (%) criteria	β (%) criteria	Suppressed connexions	Model order	
1	f_{sta}	0.31	0.10		13
2	f_{clt}	1.38	0.34		13
3	I_{wheel}	1.46	0.53	f_{tyre}	11
4	I_{em}	0.08	3.25	/	10
5	K_{sta}	2.09	0.49		8
6	K_{GB}	8.14	2.06	f_{cm}	6
7	f_{tyre}	11.58	3.25		6
8	K_{clt}	12.58	3.36		4
9	f_{eng}	27.44	9.18		4
10	f_{em}	32.17	12.38		3



$$I_{eq} = I_{sta} + I_{eng} + I_{PD} + R_{GB}^{-2} \cdot I_{ST}$$

$$K_{eq} = (K_{trans}^{-1} + K_{tyre}^{-1})^{-1} \quad T_{eq1} = \frac{T_{GB}}{T_{PS}} \quad T_{eq2} = \frac{T_{wheel}}{M_{cha}}$$

Fig. 3. Simplified model technological representation

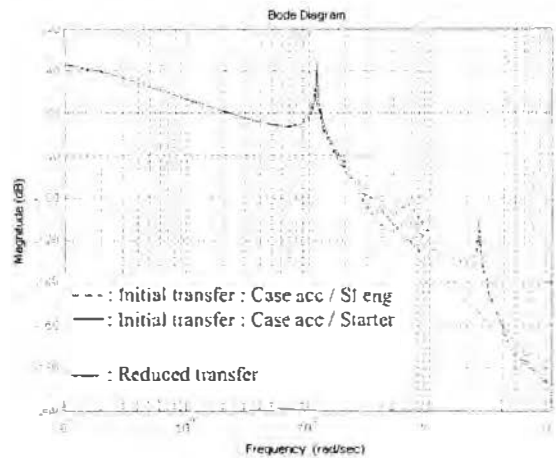


Fig. 4. Frequency responses comparison

The simplified model is obtained in a more general and systematic manner than in Hrovat and Tobler (1991). Furthermore it verifies properties empirically expected for a driveline simplified model: it is a serial structure with 2

inertias and one stiffness. The stiffness tyre is dominating and the inertia on which motors torques are applied is a concatenation of inertias directly linked to the motor inertia in the initial model.

VI. CONCLUSION AND PERSPECTIVES

Simple “physical” models are very useful for the process of understanding the design limitations. Obtaining such models from more refined one is however a difficult task. Preserving the physical meaning constrains changes of coordinates.

The methodology introduced in this paper has many interesting features. At first, it manipulates elementary physical components to get systematically minimal physical model. At second, it makes use of gramians which may be efficiently computed in order to analyze which components are of less importance. Contrary to previous physical model reduction method (Loucas, *et al.*, 1997, 1999), the state-output relation is taken into account and MIMO models can be processed. Reduction leads to a simplified physical model whose components have constitutive laws directly related to those of the initial model which is a particularly interesting property for a parametric sensitivity analysis.

So far, the proposed method has limitations. The procedure of model simplification is not fully automatic. Except for some classes of model, a minimum of expertise is necessary to perform component suppression and reorganization. Another weakness is that model error bounds are not known *a priori*. Nevertheless, the CAD software proposed in Ripert (2003) is already a valuable help for PSA-Peugeot Citroen.

Our future works will consist to i) make more explicit the algebraic rules used for model simplification, ii) generalise the approach to a larger class of model including non-linearities, iii) extend the simplification process to the connections between components.

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