



# Efficient handling of synchronization in three vehicle routing problems

Fabien Lehuédé, Philippe Grangier, Axel Grimault, Renaud Masson

## ► To cite this version:

Fabien Lehuédé, Philippe Grangier, Axel Grimault, Renaud Masson. Efficient handling of synchronization in three vehicle routing problems. *Odysseus 2015*, May 2015, Ajaccio, France. hal-01165265

**HAL Id: hal-01165265**

**<https://hal.science/hal-01165265>**

Submitted on 18 Jun 2015

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Efficient handling of synchronization in three vehicle routing problems

Fabien Lehuédé<sup>1</sup>, Philippe Grangier<sup>1,2</sup>, Axel Grimault<sup>1</sup>, Renaud Masson<sup>3</sup>

<sup>1</sup> École des Mines de Nantes, IRCCyN UMR CNRS 6597 (Institut de Recherche en Communications et Cybernétique de Nantes), 4 rue Alfred Kastler, 44307 Nantes, France

`fabien.lehuede@mines-nantes.fr`

<sup>2</sup> Department of Mathematics and Industrial Engineering and CIRRELT, École Polytechnique de Montréal and CIRRELT, C.P 6079, Succursale Centre-ville, Montréal, QC, Canada H3C 3A7

<sup>3</sup> OM-Partners, Koralenhoeve 23, 2160 Wommelgem (Antwerpen), Belgium

**Keywords:** *Vehicle routing, Synchronization constraints*

This paper presents a synthesis of contributions to the solving of three vehicle routing problems involving synchronization constraints. These problems are: the Pickup and Delivery Problem with Transfers (PDPT), studied during the PhD of Renaud Masson, co-advised with Olivier Péton [4], the Two-Echelon Multiple-Trip Vehicle Routing Problem with Satellite Synchronization (2E-MTVRP-SS), studied by Philippe Grangier during his PhD, co-advised with Michel Gendreau and Louis-Martin Rousseau [2], and the Heterogeneous Full Truckload Pickup and Delivery Problem with Time Windows and Resource Synchronization (HFTPDPWTW-RS), under study by Axel Grimault in his PhD and co-advised with Nathalie Bostel [3]. All these problems have been solved with an Adaptive Large Neighborhood Search (ALNS).

A special focus is given to the temporal feasibility evaluation of an insertion which has been proposed for the PDPT [5] and extended to the other problems. The concept of forward time slack [6] is extended to provide a constant time feasibility test of temporal constraints. Experiments confirm the solving time reduction provided by the implementation of this test in a meta-heuristic.

## 1 Problematics

Synchronization problems in vehicle routing have been recently surveyed and classified by Drexler [1]. This paper outlines the complexity raised by the interdependence between routes created by synchronization constraints. It also shows that few solution methodologies have been proposed to efficiently handle synchronization in VRP heuristics.

The PDPT involves *operation synchronization with precedences*: this problem is a generalization of the PDPTW where a request can be unloaded at a specific point, called transfer point, from where it can be loaded later in another vehicle.

The 2E-MTVRP-SS implies *exact operation synchronization*: In this city logistics problem, two fleets of vehicles are considered. A fleet of large vehicles leaves an urban distribution center to visit satellites via major roads (first echelon). At such location, goods are transferred to smaller vehicles, which drive the remaining kilometers to deliver customers within time windows (second

echelon). As no storage is allowed at satellites, a first level and a second level vehicle should be present simultaneously at a satellite during a transshipment.

The HFTPDPTW-RS involves *resource synchronization*. This problem has been met in a project with a public work company to optimize the transportation of full truckloads of gravels, asphalt, sand, ..., needed in the construction of roads. For many of these products, a specific resource (eg. a loader) is required to load or unload trucks. The service of several vehicles at one resource cannot overlap, which creates an interdependency between routes.

## 2 A general feasibility framework

In each of these problems, insertion heuristics are implemented in the ALNS framework to create or recreate routes. The incremental evaluation of the feasibility of an insertion with respect to temporal constraints is performed with the same methodology. It relies on (1) a precedence graph that models precedences on operations on routes, (2) preprocessing on the precedence graph to identify forward time slacks and slack times that encapsulate interdependences between routes, (3) a feasibility test based on preprocessed variables that is adapted to each problem. Details of the methodology can be found in [5] for the PDPT.

### 2.1 Precedence graph

Synchronization constraints are modeled for each problem on a precedence graph as illustrated on Figure 1. In these graphs, each arc has a weight that represents the minimum time between service on its origin and its destination vertex. Each vertex represents an operation, which can be delimited in time with a time window.

For the PDPT (Figure 1.a), the transfer of request 1 from route 2 to route 1 at transfer point  $\tau$  is modeled with two vertices:  $t_{\tau,1}^-$  represents the unloading of request 1 at  $\tau$  and  $t_{\tau,1}^+$  represents the pickup of this request by route 1. A precedence arc with a value equals to the duration of a transfer is created between the two transfer vertices.

Exact synchronization in the 2E-MTVRP-SS (Figure 1.b) is modeled with an entrance vertex  $T_e$  that represents the arrival of the first level vehicle at a satellite, an exit vertex  $T_x$  that represents its departure and one vertex per second level vehicle that represents the beginning of transfers in these vehicles.

Precedences due to constraints on resources in the HFTPDPTW-RS are modeled on Figure 1.c. In this case, no additional vertex is needed to model synchronization.

The precedence graph corresponds to a PERT chart. On this kind of graph (directed, acyclic), an as early as possible schedule can be computed in linear time. This schedule provides a feasible service time  $h_i$  for each vertex  $i$  in complete and partial solutions.

### 2.2 Preprocessing

The notion of forward time slacks introduced by Savelsbergh [6] is extended to incrementally evaluate the feasibility of a modification in the precedence graph.

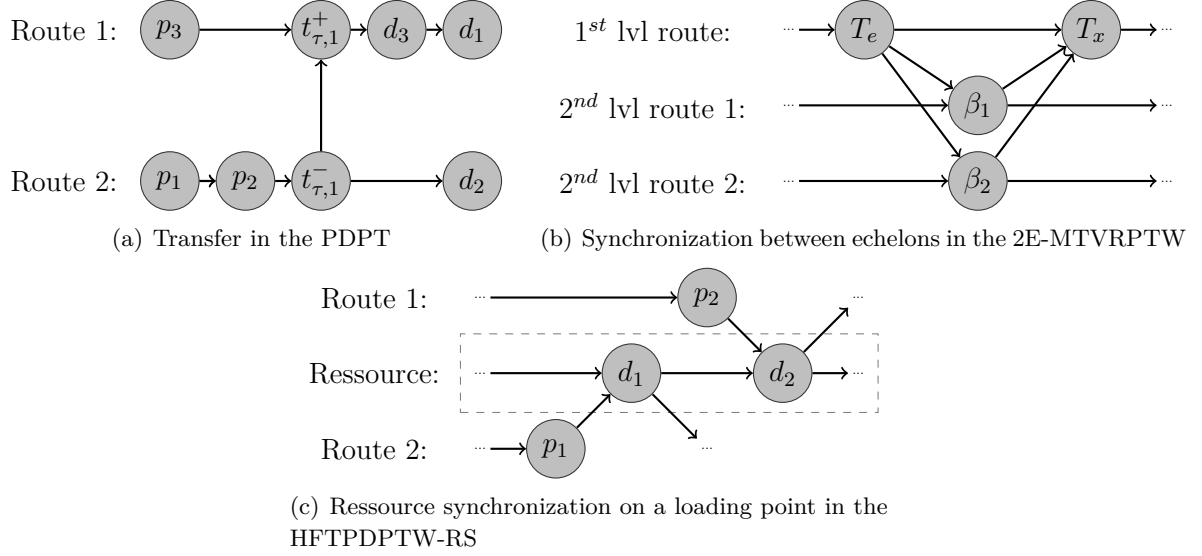


Figure 1: Synchronization modeled with precedence graphs

First, we calculate the *Slack Time*  $ST_{u,v}$  between each pair of vertex in the precedence graph:

$$ST_{u,v} = \min_{w \in \Omega_{u,v}} TWT_w,$$

where  $\Omega_{u,v}$  denotes all paths between two vertices  $u$  and  $v$  in the precedence graph and  $TWT_w$  denotes the sum of waiting times on a path  $w$ . The  $ST$  matrix is computed calculating a shortest path between all pair of vertices in the precedence graph where each arc is valued with its waiting time at destination in the schedule of the solution.

According to this definition, the forward time slack at a node  $u$  is defined as follows:

$$F_u = \min_{i \in \{u\} \cup \Gamma^+(u)} \{ST_{u,i} + l_i - h_i\}$$

Where  $h_i$  and  $l_i$  denote the time of service and the latest service time of a vertex  $i$ , respectively and  $\Gamma^+(u)$  denotes the set of successors of vertex  $u$  in the precedence graph.

In addition, for each pair of vertex  $(i, j)$  in the precedence graph, preprocessing calculates the boolean value  $\Gamma^+(i, j)$  if vertex  $j$  is a successor of  $i$  in this graph.

If a path exists in the precedence graph between a vertex  $u$  and a vertex  $v$ , delaying by  $\delta$  minutes the service at  $u$  implies a delay of  $\max(\delta - ST_{u,v}, 0)$  minutes at  $v$ . The forward time slack  $F_u$  represents the maximum value by which the service at a vertex  $u$  can be delayed without causing the violation of the time window of one of its successors in the precedence graph. The computation of these variables can be done in quadratic time in the size of the precedence graph [5].

### 2.3 Feasibility test

The insertion of a request in a partial solution creates new vertices and arcs in the precedence graph. Numerous insertions are evaluated in terms of feasibility and cost without being performed. While different for each problem and each insertion type, the feasibility evaluation follows the same two steps:

First, a request insertion may create a cycle in the precedence graph. A constant time test based on the  $\Gamma^+$  matrix can be performed to identify newly created successor relationships in the precedence graph. If an insertion implies that a vertex  $i$  becomes a successor of a vertex  $j$  and  $\Gamma^+(i, j)$  is true, then this insertion creates a cycle and is not feasible.

Second, the feasibility of a delay on the service at one vertex in the precedence graph can be assessed with its forward time slack without having to reschedule its successors. Complex insertions such as the insertion of a pickup and delivery request with a transfer in the PDPT can be efficiently evaluated using slack times to calculate the impact of a delay of one vertex on all its successors.

### 3 Conclusion

The impact of the constant time feasibility test has been evaluated in [2], comparing an incremental implementation of the PERT algorithm with the proposed approach in the 2E-MTVRP-SS. As shown on Table 1 the feasibility test is on average 12 times faster than the selective rescheduling of a solution for each evaluated insertion.

Instance	c101	c102	c201	c202	r101	r102	r201	r202	rc101	rc102	rc201	rc202	average
PERT (in min)	772.4	696.5	1043.2	933.1	483.5	244.0	292.8	543.1	598.4	804.3	456.0	1612.6	706.7
FTS (in min)	<b>55.5</b>	<b>74.1</b>	<b>67.6</b>	<b>80.7</b>	<b>34.7</b>	<b>35.7</b>	<b>35.4</b>	<b>54.6</b>	<b>38.4</b>	<b>54.9</b>	<b>43.1</b>	<b>85.7</b>	<b>55.0</b>
Difference (in %)	-92.8	-89.4	-93.5	-90.9	-92.8	-85.4	-87.9	-89.9	-93.6	-93.2	-90.5	-94.7	-92.2

Table 1: Run times of incremental PERT versus FTS for 25 000 iterations of the ALNS for the 2E-MTVRP-SS

To conclude, the proposed framework significantly improves the solving of the studied problems. We believe that it can be applied to other VRP with synchronization constraints.

## References

- [1] M. Drexl. Synchronization in Vehicle Routing—A Survey of VRPs with Multiple Synchronization Constraints. *Transportation Science*, 46(3):297–316, March 2012.
- [2] Philippe Grangier, Michel Gendreau, Fabien Lehuédé, and Louis-martin Rousseau. An Adaptive Large Neighborhood Search for the Two-Echelon Multiple-Trip Vehicle Routing Problem with Satellite Synchronization. Technical Report July, Les cahiers du CIRRELT - CIRRELT-2014-33, 2014.
- [3] Axel Grimault, Fabien Lehuédé, and Nathalie Bostel. A two-phase heuristic for full truckload routing and scheduling with split delivery and resource synchronization in public works. In *2014 International Conference on Logistics Operations Management*, pages 57–61. IEEE, June 2014.
- [4] Renaud Masson, Fabien Lehuédé, and Olivier Péton. An Adaptive Large Neighborhood Search for the Pickup and Delivery Problem with Transfers. *Transportation Science*, 47(August):344–355, September 2013.
- [5] Renaud Masson, Fabien Lehuédé, and Olivier Péton. Efficient feasibility testing for request insertion in the Pickup and Delivery Problem with Transfers. *Operations Research Letters*, 41(3):211–215, September 2013.
- [6] M.W.P. Savelsbergh. Local search in routing problems with time windows. *Annals of Operations Research*, 4(1):285–305, 1985.